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STEADY RADIAL GAS FLOW THROUGH
POROUS MEDIA

by



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A THESIS

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled STEADY RADIAL GAS FLOW THROUGH POROUS MEDIA submitted by Arun Ramchandra Piplasure in partial fulfilment of the requirements for the degree of Master of Science.

ABSTRACT

Recent gas flow studies on linear systems noted the importance of the interaction of molecular streaming and inertial effects. This led to speculation regarding the nature of these effects in radial systems. As a result, it was decided to examine radial flow both experimentally and theoretically.

Radial gas flow theory has been modified to account for viscous, inertial, and molecular streaming effects. In order to carry out the experimental verification of the proposed theory, flow tests were conducted on three Indiana limestone samples measuring approximately one foot in diameter and one to one and a half inches in thickness. A radial flow cell was specially designed, built, and tested for the purpose.

The resulting data, when examined both graphically and numerically, indicated that all three effects are important, and that the suggested modification of the graphical procedure leads to more realistic parameter estimations. It has also been found that the Klinkenberg graphical method is inapplicable under certain conditions, and that the manner in which a test sample is confined during a laboratory flow test is a factor of considerable importance.

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INTRODUCTION

The phenomenon of gas flow through porous media has been a subject of investigation for quite some time. Numerous investigators have carried out experiments to verify Darcy's equation for viscous flow and Forchheimer's modification of it for high velocity flow. The phenomenon of gas-slippage,* studied by Klinkenberg, has also been experimentally verified in many cases.

Considerable research has been done on these subjects at the University of Alberta and many interesting results obtained. However, the majority of this work and that done elsewhere has been on linear systems.

In general, the flow in natural gas reservoirs is roughly radial. Hence, the question arises whether the results obtained on linear systems may be applied to the actual radial reservoirs by a mere transformation of geometry, or whether some factors hitherto unaccounted for come into play.

Because of the relative scarcity of results on radial systems, this study was undertaken. From a theoretical standpoint this necessitated a reconsideration of the basic theory of the flow of gases in porous media, especially in light of the recent studies on linear systems which empha-

* the terms gas-slippage and molecular streaming are used interchangeably in this work.

size the importance of molecular streaming. For carrying out the experimental verifications, new equipment was designed and assembled to carry out the gas flow tests.

LITERATURE REVIEW

The available literature on the subject of fluid flow through porous media is quite abundant. To encompass the various aspects of this topic, the present section is sub-classified to review the work done in the areas of viscous flow, visco-inertial, flow and their experimental verifications.

Viscous Flow

The experiments of Henry Darcy (1856), concerning the flow of water through sand filters, was the first scientific study of fluid flow through porous media. As a result of this and further investigations the following relationship, known as Darcy's law, was proposed to describe steady horizontal flow of fluids in the viscous region.

$$-\frac{dp}{dx} = \frac{\mu q}{k} \quad (1)$$

M. King Hubbert ⁽¹⁾ has shown the theoretical soundness of this experimental proposition by deriving it from the fundamental Navier-Stokes equation of motion for viscous fluids.

Although equation (1) holds in case of liquid flow, it was shown by Klinkenberg ⁽²⁾ that the permeability constant as determined with gases is dependent upon the nature of

the gas, and is approximately a linear function of the reciprocal mean pressure. This phenomenon was attributed to molecular streaming. In view of this, the permeability k in equation (1) should be replaced by an apparent permeability k_a defined as

$$k_a = k \left(1 + \frac{b}{p} \right) \quad (2)$$

Under the assumptions of isothermal, viscous flow of gas in a homogeneous medium, Al-Hussainy et al. ⁽³⁾ studied the effect of variations of pressure-dependent gas viscosity and compressibility. For steady radial gas flow in the viscous region they proposed the relation

$$q_{sc} = \frac{\pi k h T_{sc} [m(p_e) - m(p_w)]}{T p_{sc} \ln \frac{r_e}{r_w}} \quad (3)$$

where

$$m(p) = 2 \int_{p_m}^p \frac{p}{\mu(p) Z(p)} dp \quad (4)$$

Visco-Inertial Flow

It was observed that, when the fluid flow rate was continuously increased, Darcy's law did not adequately describe the flow behaviour beyond a certain flow rate. To describe this high velocity flow, Forchheimer⁽⁴⁾ suggested a modification of Darcy's law and proposed an equation

$$\frac{\Delta p}{\Delta x} = aq + bq^2 \quad (5)$$

where a and b are thought to be constants.

This model was obtained on the basis of empirical reasoning and curve fitting techniques applied to various experimental data when available.

Adopting an approach similar to that of M. King Hubbert,⁽⁵⁾ Chwyl⁽⁶⁾ derived Forchheimer's equation from the fundamental Navier-Stokes equation, thereby putting it on a sound theoretical basis.

Green and Duwez⁽⁷⁾ demonstrated the validity of the equation

$$-\frac{dp}{dx} = \alpha\mu q + F_B\rho q^2 \quad (6)$$

for describing high velocity linear flow. On the basis of this, they defined the Reynold's number for porous medium as

$$Nr_e = \frac{F_B \rho q}{\alpha \mu} \quad (7)$$

where

$$\alpha = \frac{1}{k}$$

and F_B = the inertial resistance coefficient.

Using the modified gas law as an equation of state, Greenberg and Weger⁽⁸⁾ integrated equation (6) to yield

$$\frac{\Delta(p^2)Mg}{2L(ZRT)\mu G} = + F_B \left(\frac{G}{\mu}\right) \quad (8)$$

where $G = \rho q$ = gas mass velocity

and $g = 32.17 \text{ lbm-ft/lbf-sec}^2$.

Defining Reynold's number as

$$Nr_e = \frac{Dvp}{\mu X^n}$$

and the Fanning friction factor as

$$\frac{dp}{dr} = \frac{fv^2\rho}{2gDX^n}$$

and utilizing the friction-factor vs. Reynolds number charts for Brownell and Katz, the following integrated forms for radial systems for laminar and turbulent regions were given

by Elenbaas and Katz; (9)

$$(p_e^2 - p_w^2) = 0.000812 \frac{\bar{\mu} \bar{Z} T W}{D^2 \phi^{n-m} h G} \ln \frac{r_e}{r_w} \quad (9)$$

$$(p_e^2 - p_w^2) = 0.00000202 \frac{W^2 \bar{T} \bar{Z} f}{D^2 n_h^2 G} \left(\frac{1}{r_w} - \frac{1}{r_e} \right) \quad (10)$$

where W = mass flow rate in lbs/sec

D = average diameter of particles in ft

ϕ = porosity of the bed

h = bed thickness in ft

G = gass gravity

and n and m are the exponents to be determined as suggested in their paper.

Combining the modified gas law with the radial equivalent of equation (6), Tek et al. (10) presented the following integrated equation,

$$p_e^2 - p_w^2 = \frac{1424 \bar{\mu} \bar{Z} T Q_0 \ln \left(\frac{r_e}{r_w} \right)}{h k} + \frac{3.1602 \times 10^{-12} F_B \bar{G} \bar{T} \bar{Z} Q_0^2 \left(\frac{1}{r_w} - \frac{1}{r_e} \right)}{h^2} \quad (11)$$

where $\bar{\mu}$ = viscosity in centipoise evaluated at arithmetic mean temperature and pressure

$\bar{Z}$ = compressibility factor evaluated at arithmetic mean temperature and pressure

$\bar{T}$ = average temperature of the flowing gas, °R

Q_0 = volumetric flow rate, MSCF/day

k = permeability in millidarcies

and F_B = inertial resistance coefficient, ft^{-1} .

This so far is the most widely accepted model for steady radial gas flow.

Assuming porous media to be made up of bundles of capillary tubes, with orifice plates spaced throughout the tubes at a distance equal to the tube diameter, Blick⁽¹¹⁾ proposed a capillary orifice model. Applying the force momentum balance and modified gas law, he derived the following equation for describing steady linear isothermal gas flow,

$$\frac{(p_1^2 - p_2^2) M \delta \phi^2}{2 L G^2 Z R T} = 2 C_f + \frac{C_D}{2} \quad (12)$$

where $C_f = \frac{16}{N r_e}$, for viscous flow

and $\frac{1}{\sqrt{C_f}} = 4.0 \log_{10} (N r_e \sqrt{C_f}) - 0.4$, for turbulent regime

C_D = drag coefficient of orifice plate

$G = \rho v$

δ = average pore diameter, ft.

and ϕ = porosity fraction.

As a result of this development, he was able to develop the following theoretical expression for the inertial resistance coefficient:

$$F_B = \frac{\rho C_D}{2A\phi^2\delta} \quad (13)$$

where $A \approx 1.0$.

Extending Blick's⁽¹²⁾ technique to radial systems, Crafton⁽¹³⁾ suggested the following equation for radial gas flow:

$$\begin{aligned} (p_e^2 - p_w^2) = & 18546 \frac{Q_w \rho_w \mu ZT}{khG} \left(\ln \frac{r_e}{r_w} - \frac{1}{2} \right) \\ & + 5.41 \times 10^{-10} \frac{F_B Q_w^2 \rho_w^2 ZT}{h^2 G} \left(\frac{1}{r_w} - \frac{4}{3r_e} \right) \end{aligned} \quad (14)$$

where Q_w = well bore flow rate, MCF/day

ρ_w = well bore fluid density, lbm/ft³.

Experimental Investigations

Many experimental investigations have been carried out to verify the validity of Darcy's law for gas flow through porous media and Klinkenberg's slippage phenomenon. Notable

among these are the works of Estes and Fulton,⁽¹⁴⁾ Rose,⁽¹⁵⁾ Grunberg and Nissan,⁽¹⁶⁾ and Calhoun and Yuster.⁽¹⁷⁾

On the basis of flow experiments conducted on porous sintered metals, Greenberg and Weger⁽¹⁸⁾ reported that permeability, while not a function of pressure, varied inversely with temperature. They concluded that the inertial resistance coefficient remained constant for pressures up to 2000 psia in the temperature range 15°C to 60°C.

Stewart and Owens⁽¹⁹⁾ studied the extent to which slippage and inertial effects could exist simultaneously, by performing tests on heterogeneous limestones. Though at very large Reynold's numbers the slippage effects disappeared, it was found that both slippage and inertial effects were present at high flow rates.

Dranchuk and Sadiq⁽²⁰⁾ have shown the need for proper delineation of the viscous and visco-inertial regions, so that the Klinkenberg plot of equations (2) and the visco-inertial plot of equation (8) give consistent permeability values.

Hamilton⁽²¹⁾ considered the slippage to be significant at low mean pressures. Using equation (8) he obtained the values of k and F_B from the straight line portion of the visco-inertial plot. The downwarping of the plot at lower flow rates was considered to be due to slippage effects.

Investigating the magnitude of resulting errors, when average values for viscosity and compressibility of the fluid

are used, Mackett⁽²²⁾ concluded that the values of the apparent gas permeability calculated using these properties did not differ appreciably from the values obtained using integrals to express the isothermal variation of these properties.

While re-examining the data of previous investigators, Kolada⁽²³⁾ observed that invariably the permeability obtained from a Klinkenberg plot was lower than that given by a standard visco-inertial plot. This discrepancy was attributed to molecular streaming effects which had been neglected in the visco-inertial flow theory. Considering molecular streaming for flow under both low and high pressure gradients, he developed a modified form of the visco-inertial gas flow theory for linear systems and also presented its experimental verification.

Based on the above work, Dranchuk and Kolada⁽²⁴⁾ have suggested a modified technique for interpreting laboratory data obtained in gas flow tests conducted on linear porous media.

STEADY RADIAL GAS FLOW WITH SLIPPAGE

The work of Dranchuk and Kolada⁽²⁵⁾ on linear systems has established the need for incorporating molecular streaming effects into the basic visco-inertial gas flow theory.

As seen in the linear case, the molecular streaming effects being important, their consideration in the radial visco-inertial gas flow theory is warranted. Examination of the available models of Tek et al.⁽²⁶⁾ and of Crafton⁽²⁷⁾ shows that they do not allow for this effect. However, it appears that they could be modified in order to account for gas slippage. In fact, the Tek et al.⁽²⁸⁾ model was so modified. Modification of Crafton's⁽²⁹⁾ model though planned, was not completed due to shortage of time.

A Modified Radial Gas Flow Equation

The modification of the Tek et al.⁽³⁰⁾ model may be explained by means of the following reasoning.

Steady radial horizontal gas flow under the conditions of no gas slippage is described by the relationship

$$-\frac{dp}{dr} = \frac{\mu q}{k} + F_B \rho q |q| \quad (15)$$

In this equation, the absolute value of 'q' is employed be-

cause 'q' is conventionally taken as positive when flow is in the direction of increasing 'r' and negative when it is in the opposite direction.

Using the modified gas law together with average values of viscosity, compressibility and temperature, the above equation on integration yields the following form which is essentially the same as that suggested by Tek et al. (31) except that the sign convention for 'q' or 'Q₀' has been used.

$$(p_e^2 - p_w^2) = - \left[\frac{1424 \bar{\mu} \bar{Z} \bar{T} Q_0 \ln\left(\frac{r_e}{r_w}\right)}{hk} + \frac{3.1602 \times 10^{-12} F_B \bar{G} \bar{T} \bar{Z} Q_0 |Q_0| \left(\frac{1}{r_w} - \frac{1}{r_e}\right)}{h^2} \right] \quad (16)$$

Under the conditions of gas slippage, equation (15) has to be modified so that,

$$- \frac{dp}{dr} = \frac{\mu q}{k_a} + F_B \rho q |q| \quad (17)$$

Multiplying through by ρ yields

$$- \rho \frac{dp}{dr} = \frac{\mu q \rho}{k_a} + F_B \rho^2 q |q| \quad (18)$$

By virtue of the modified gas law,

$$\rho = \frac{pM}{ZRT} \quad (19)$$

and

$$\rho_0 = \frac{p_0 M}{RT_0} \quad (20)$$

Furthermore, for plane radial flow,

$$q = \frac{Q}{A} = \frac{Q}{2\pi rh} \quad (21)$$

and

$$\rho_0 Q_0 = \rho Q \quad (22)$$

Using the average values of viscosity and compressibility evaluated at the arithmetic mean temperature and pressure and substituting equations (19), (20), (21), and (22) into equation (18) yields,

$$\begin{aligned} - \frac{M}{ZRT} \frac{p dp}{dr} &= \frac{p_0 Q_0 M \bar{\mu}}{RT_0 k_a (2\pi rh)} \\ &+ F_B \left(\frac{p_0 M}{RT_0} \right)^2 \frac{Q_0 |Q_0|}{(2\pi rh)^2} \end{aligned} \quad (23)$$

or

$$- p \frac{dp}{dr} = \frac{\overline{\mu Z T p_0 Q_0}}{k_a T_0 (2\pi r h)} + F_B \left(\frac{p_0^M}{RT_0} \right)^2 \frac{\overline{Z R T}}{M} \frac{Q_0 |Q_0|}{(2\pi r h)^2} \quad (24)$$

but

$$k_a = k \left(1 + \frac{b}{p} \right) \quad (25)$$

Hence

$$- p \frac{dp}{dr} = \frac{\overline{\mu Z T p_0 Q_0}}{k \left(1 + \frac{b}{p} \right) T_0 (2\pi r h)} + F_B \left(\frac{p_0^M}{RT_0} \right)^2 \frac{\overline{Z R T}}{M} \frac{Q_0 |Q_0|}{(2\pi r h)^2} \quad (26)$$

Defining

$$C_1 = \frac{\overline{\mu Z T p_0 Q_0}}{k T_0 (2\pi h)} \quad (27)$$

and

$$C_2 = F_B \left(\frac{p_0^M}{RT_0} \right)^2 \frac{\bar{Z} \bar{R} \bar{T}}{M} \frac{Q_0 |Q_0|}{(2\pi h)^2} \quad (28)$$

equation (26) can be concisely written as

$$-p \frac{dp}{dr} = \frac{C_1}{\left(1 + \frac{b}{p}\right)r} + \frac{C_2}{r^2} \quad (29)$$

which may also be written as

$$-p \frac{dp}{dr} = \frac{C_1 r + C_2 \left(1 + \frac{b}{p}\right)}{\left(1 + \frac{b}{p}\right)r^2} \quad (30)$$

Equation (30) is a non-linear ordinary differential equation for which there is as yet no known solution. However, if $\frac{b}{p}$ is sufficiently small, this equation may be approximated by means of the equation

$$-p \frac{dp}{dr} = \frac{C_1 r + C_2}{\left(1 + \frac{b}{p}\right)r^2} \quad (31)$$

which is separable. Equation (31) may be written as

$$-p \left(1 + \frac{b}{p}\right) dp = \left(\frac{C_1}{r} + \frac{C_2}{r^2}\right) dr \quad (32)$$

and integrated between the limits,

$$r = r_w, \quad p = p_w$$

$$r = r_e, \quad p = p_e$$

to give

$$- \left[\frac{(p_e^2 - p_w^2)}{2} + b(p_e - p_w) \right] = C_1 \ln\left(\frac{r_e}{r_w}\right) + C_2 \left(\frac{1}{r_w} - \frac{1}{r_e} \right) \quad (33)$$

When multiplied throughout by -2.0 this equation becomes

$$(p_e^2 - p_w^2) + 2b(p_e - p_w) = -2 \left[C_1 \ln\left(\frac{r_e}{r_w}\right) + C_2 \left(\frac{1}{r_w} - \frac{1}{r_e} \right) \right] \quad (34)$$

If the customary field units of

$$\mu = \text{centipoise}, \quad h, r_w, r_e = \text{ft},$$

$$k = \text{millidarcy}, \quad Q_0 = \text{MSCF/day},$$

$$T_0 = 520^\circ \text{R}, \quad p_0 = 14.7 \text{ psia}$$

and G = gas gravity, where gravity of air = 1.0 are employed, this equation takes the form,

$$(p_e^2 - p_w^2) + 2b(p_e - p_w) = - \left[\frac{1424 \bar{\mu} \bar{Z} \bar{T} Q_0 \ln\left(\frac{r_e}{r_w}\right)}{hk} \right. \quad (35)$$

$$\left. + \frac{3.1602 \times 10^{-12} F_B \bar{G} \bar{T} \bar{Z} Q_0 |Q_0| \left(\frac{1}{r_w} - \frac{1}{r_e}\right)}{h^2} \right]$$

It is interesting to note that, although equation (35) is but an approximate solution to the differential equation describing flow where slippage, viscous, and inertial effects are important, neglecting inertial effects causes the equation to collapse to the form

$$(p_e^2 - p_w^2) + 2b(p_e - p_w) = - \left[\frac{1424 \bar{\mu} \bar{Z} \bar{T} Q_0 \ln\left(\frac{r_e}{r_w}\right)}{hk} \right] \quad (36)$$

which is in fact the rigorous solution for that case.

USE OF THE PROPOSED EQUATIONS FOR
PARAMETER ESTIMATIONS

The forms of both the equations (35) and (36) are such that they are quite amenable to graphical treatment. However, in case of equation (35), one of the three parameters must be either known or assumed in order to solve for the other two. For example, if 'b' is known a priori, 'k' and 'F_B' may be evaluated.

In case of flow towards the well bore, rearrangement of equation (36) after multiplication by $k/(p_e^2 - p_w^2)$ gives

$$k \left(1 + \frac{b}{p}\right) = \frac{1424 \bar{\mu} \bar{Z} \bar{T} Q_0 \ln\left(\frac{r_e}{r_w}\right)}{(p_e^2 - p_w^2)h} \quad (37)$$

It can be recognized that the right hand side of the above equation is in fact the apparent permeability k_a . Hence, it along with $\frac{1}{p}$ may be used to construct the Klinkenberg plot for the radial flow case, from which 'k' and 'b' may be determined.

As observed by Dranchuk and Sadiq,⁽³²⁾ the Klinkenberg plot gives meaningful results provided the data used are in the viscous region. This presents the difficulty of delineating the viscous and visco-inertial flow regions. By employing the Reynold's number criterion, this difficulty can

be overcome. Since the Reynold's number for a porous medium is defined as the ratio of inertial effects to viscous effects, it can be obtained by taking the ratio of the second term to the first term on the right hand side of equation (35).

It may be observed that equation (35) can be rearranged to yield

$$- \left[\frac{[(p_e^2 - p_w^2) + 2b(p_e - p_w)]}{1424 \bar{\mu} \bar{Z} \bar{T} Q_0} \right] = \left[\frac{\ln\left(\frac{r_e}{r_w}\right)}{k} + \frac{2.2192 \times 10^{-15} F_B G \left(\frac{1}{r_w} - \frac{1}{r_e}\right) \left(\frac{|Q_0|}{\bar{\mu}}\right)}{h} \right] \quad (38)$$

When the value of 'b' obtained in the previous plot is used, equation (38) suggests a straight line plot whose intercept and slope should yield the values of 'k' and 'F_B' respectively. On the other hand, equation (16) may be placed in the following form

$$- \left[\frac{(p_e^2 - p_w^2)h}{1424 \bar{\mu} \bar{Z} \bar{T} Q_0} \right] = \left[\frac{\ln\left(\frac{r_e}{r_w}\right)}{k} + \frac{2.2192 \times 10^{-15} F_B G \left(\frac{1}{r_w} - \frac{1}{r_e}\right) \left(\frac{|Q_0|}{\bar{\mu}}\right)}{h} \right] \quad (39)$$

which may also be plotted in order to estimate 'k' and 'F_B'. However, in this case, the line passing through high flow rate data points which presumably lie on a straight line must be extrapolated.

In view of the fact that the plot made employing equation (38) is the radial flow equivalent of Kolada's⁽³³⁾ modified visco-inertial plot, and the plot made employing equation (39) corresponds to his conventional visco-inertial plot; these two plots are referred to as the modified and conventional visco-inertial plots respectively.

Since these graphical methods are very similar to those employed by Kolada⁽³⁴⁾ for the linear case, they pose similar problems. For example, the curvature in the conventional visco-inertial plot makes the delineation of the straight line portion of the curve difficult if not impossible. On the other hand, determination of 'b' from the Klinkenberg plot and 'k' and ' F_B ' from the modified visco-inertial plot requires data-splitting. This in turn requires not only a large number of data points but approximately the same number of points for each plot if the 'k' values obtained from the two plots are to be consistent. Furthermore, since data splitting is based on the Reynold's number criterion, the delineation of the viscous region is not only a trial and error procedure, but it is also dependent upon the Reynold's number one chooses.

From this it is obvious that a numerical technique, which utilizes all of the data simultaneously and does not require the delineation of flow regimes, would be preferable to the graphical methods. One such technique is that of quasilinearization suggested by Kolada.⁽³⁵⁾

EXPERIMENTAL APPARATUS

In order that plane radial gas flow through porous media be examined experimentally, samples of Indiana limestone, resembling large grind stones, were obtained from Imperial Oil Production Research Laboratory in Calgary. These samples were about one foot in diameter and between one and two inches thick.

Prior to flow tests, the porosity of these samples was determined using a specially constructed Boyle's law porosimeter. This essentially consisted of a core holder and an expansion chamber connected to gas supply and a vacuum pump as schematically shown in Figure 1. By calibrating with solid aluminium blanks of known dimensions, the gross grain volume of the cores was estimated and porosity determined.

In order to conduct flow tests on the available samples, a special core holder, as described below, was designed and constructed.

Core Holder

A mild steel cylinder having an outer diameter of 18 inches, a height of 3.862 inches, and a wall thickness of 2.50 inches was placed between two mild steel blind flanges which were 22 inches in diameter and 2.75 inches thick as shown in Figure 2. A seal between the three pieces was

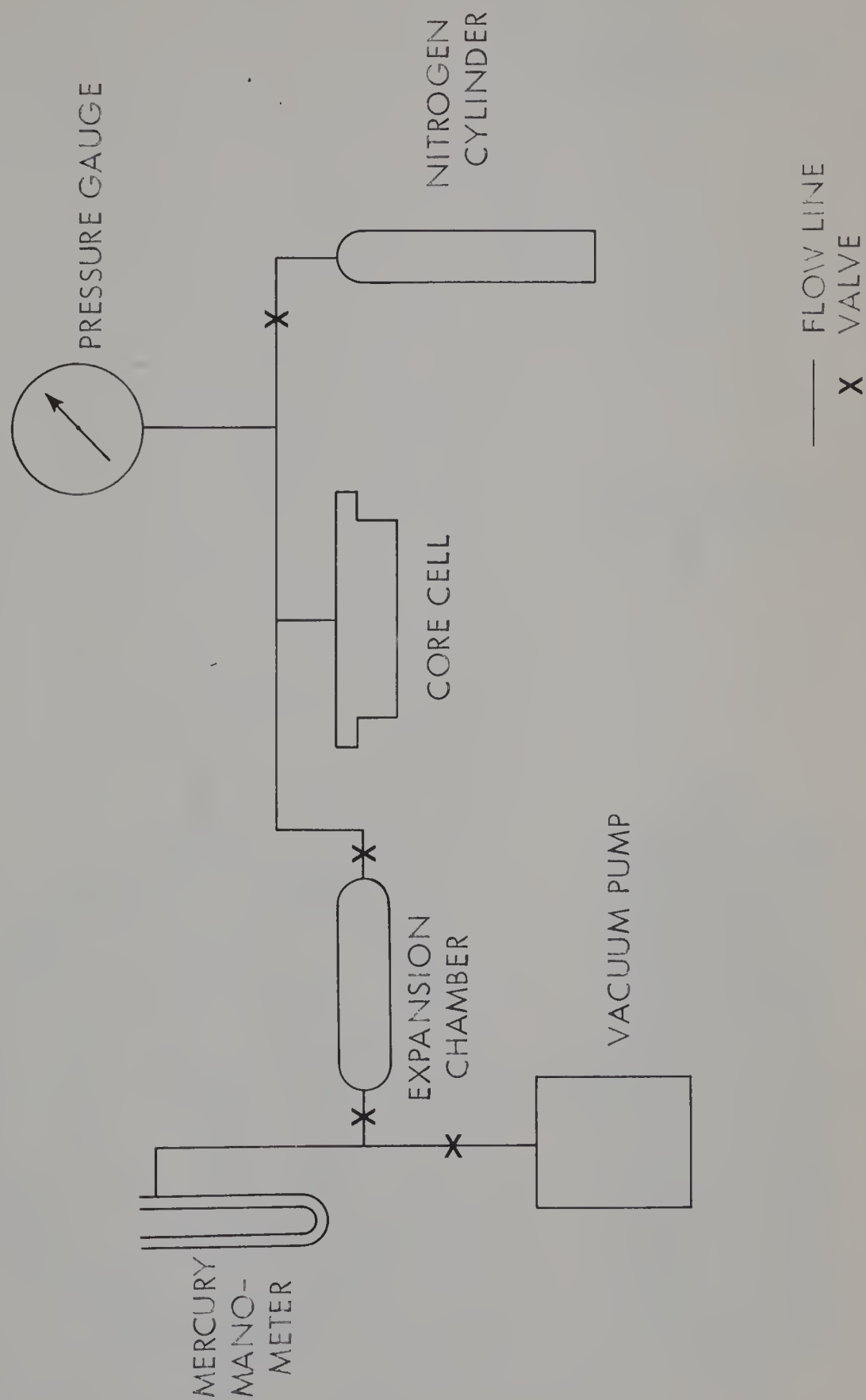


FIGURE 1. FLOW DIAGRAM OF THE EQUIPMENT FOR POROSITY DETERMINATION

effected by means of 'O' ring grooves and 'O' rings positioned in the top and bottom of the cylinder. A close fitting piston was placed inside the cylinder. The piston was constructed of mild steel, was 1.778 inches thick, and had an 'O' ring and two back up rings in its periphery. This piston could be raised or lowered hydraulically, by means of an Owattona series y21-1 model 'C' hydraulic pump. Four $\frac{3}{8}$ inch holes were drilled around the circumference of the cylinder to serve as gas inlets, while a central hole in the top flange was provided to serve as the gas outlet. In preparation for a run, the sample was placed on the piston between two $\frac{1}{8}$ inch thick natural rubber gaskets cut to the exact shape of the rock samples. The blind flanges were then bolted together by means of ten equiangularly placed one inch thick bolts. The piston was raised until the rock sample was confined between it and the upper flange.

Apparatus Assembly

The apparatus assembly is schematically shown in Figure 3. The four gas inlets were connected to a common manifold constructed of one inch pipes. The manifold in turn was connected to the gas supply cylinder via regulators R_1 and R_2 which were used for controlling low and high upstream pressures respectively. The gas outlet was connected to a soap film burette and wet test meter which were used for measuring low and high flow rates respectively. The wet test

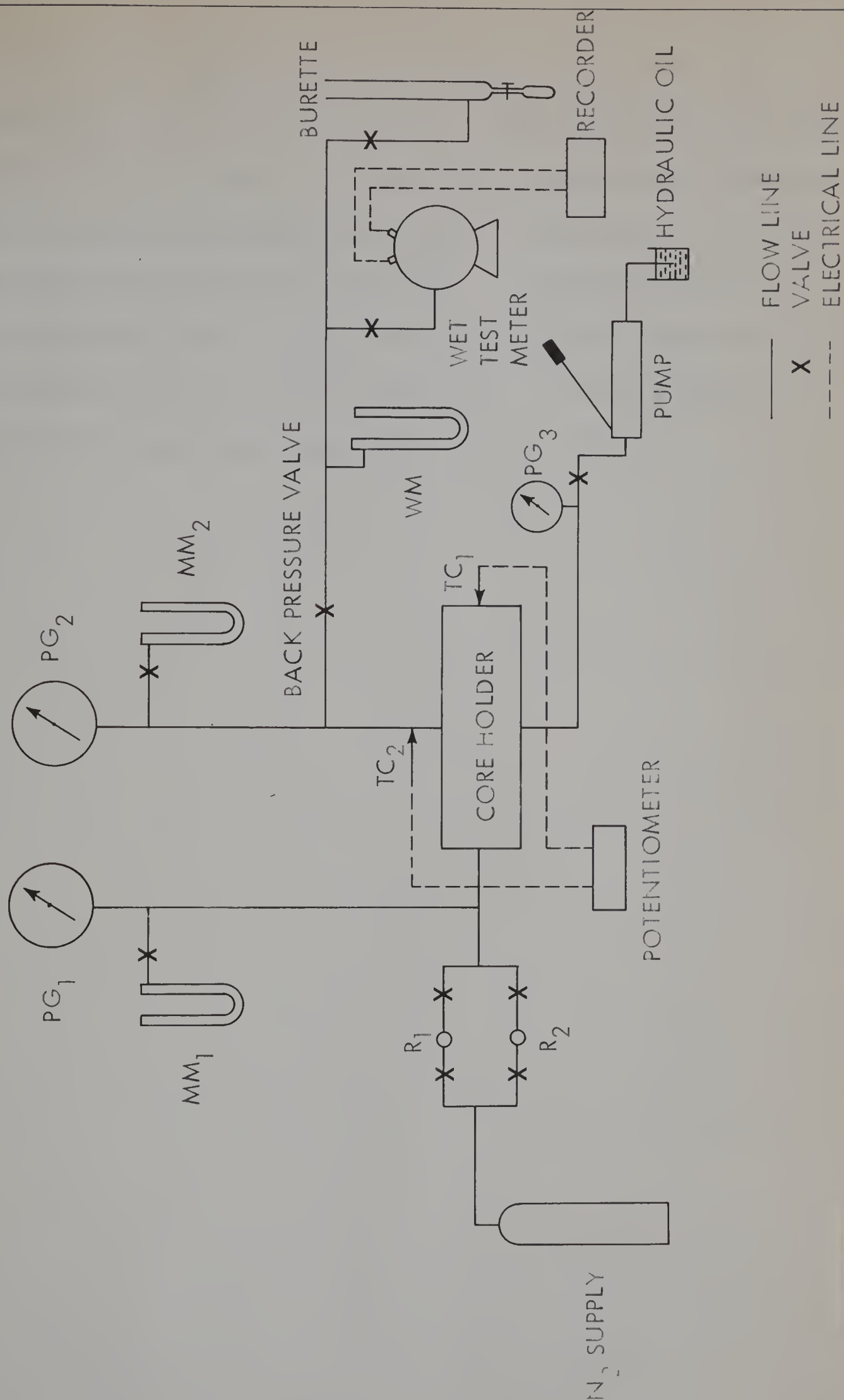


FIGURE 3. FLOW DIAGRAM FOR ASSEMBLED EQUIPMENT FOR GAS FLOW

meter was in turn connected to an 'Angus' automatic strip chart recorder with an event marker. While the mercury manometer MM_1 and Heise pressure gauge PG_1 recorded the upstream pressure, the mercury manometer MM_2 and the pressure gauge PG_2 recorded the downstream pressure. Copper-constantan thermocouples TC_1 and TC_2 were used to measure the inlet and outlet temperatures of the flowing gas. Pressure gauge PG_3 indicated the confinement pressure on the sample.

ROCK SAMPLES AND FLOW TEST METHODS

A total of three samples of Indiana limestone were studied. Two of these were obtained from the Imperial Oil Research Laboratory in Calgary. The third was cut in the workshop of the Department of Chemical and Petroleum Engineering, University of Alberta, from Indiana limestone slabs supplied by Sharp Bros. Cut Stone Company, Hamilton, Ontario.

The samples obtained from the Imperial Oil Laboratory were somewhat more heterogeneous than the one prepared at the University, since they had been acid treated in the vicinity of the inner radius before being subjected to the flow studies described herein.

Employing a combination of one of two different methods of maintaining confining pressure on the sample and one of two different ways of changing the flow rate, a total of four different types of flow tests were conducted. These tests were conducted in the following manner.

- (1) The confining pressure was held constant throughout the test and flow rate was changed by raising the upstream pressure while holding the downstream pressure at atmospheric.
- (2) The sample was confined as in (1) above and the flow rate was increased in stepwise fashion by decreasing the downstream pressure while the upstream pressure was held constant at a certain fixed value.

- (3) The flow rate variation technique was identical to that in (1) above but the confining pressure was increased with each flow rate increase so that the difference between the confining pressure and the arithmetic mean flowing pressure was held constant at some preselected value.
- (4) A combination of the flow rate variation technique employed in (2) and the confining pressure technique employed in (3) was used.

EXPERIMENTAL PROCEDURE

The pressure gauges, wet test meter, and thermocouples were calibrated before connecting them to the experimental apparatus. All the fittings and connections were pressure tested to ensure a leak-proof assembly.

The outer diameter, inner diameter, and thickness were obtained for each sample by taking the average of ten caliper measurements. Bulk volumes were calculated using these average values. Porosity was determined using the equipment previously described.

The sample was properly mounted inside the sample holder and subjected to the desired confining pressure.

Gas from the supply cylinder was flowed through the appropriate regulator at the desired pressure, and was allowed to flow for a sufficient time to ensure stabilized conditions. Inlet and outlet pressures and temperatures and corresponding flow rate measurements were taken using the appropriate devices. Barometric pressure, temperature of the flowing gas prior to entering the flow devices, and the pressure shown by water manometer ('WM' in Figure 3) were also recorded.

TREATMENT OF DATA

Volumetric flow rates of Nitrogen at the recorded temperatures and pressures (given by manometer 'WM') were converted to the reference conditions of one international atmosphere (14.696 Psia) and 520°R, using the modified gas law. Compressibility factors at required temperatures were determined by five-point Lagrangian interpolation of the curve fitted data of Hilsenrath et al. (36)

Viscosity values at the arithmetic mean temperature and pressure of the flowing gas were obtained using the following relationship suggested by Kestin and Wang. (37)

$$\begin{aligned} \mu = & 1778 \times 10^{-7} (1 + 8.958 \times 10^{-4} (p-1) \\ & + 6.120 \times 10^{-7} (p-1)^2 + 3.997 \times 10^{-8} (p-1)^3) \\ & + 4.55 \times 10^{-7} (T-25) \end{aligned} \quad (40)$$

where μ is in poise

p is in international atmospheres

and T is in °C.

A computer program was written to evaluate the expressions necessary for making the Klinkenberg permeability plot,

the conventional visco-inertial plot, and the modified visco-inertial plot. Listings of this program are given in Appendix C. A program was also written to perform a linear least squares fit of the data wherever needed.

The parameters k , b , and F_B were evaluated by applying quasilinearization to equation (35). The technique of quasilinearization has been discussed in detail by Kolada.⁽³⁸⁾ The salient points of quasilinearization as applied in the present case, are given in Appendix A. A sample of the computer program, written to effect the quasilinearization solution, is given in Appendix C.

Quasilinearization being an iterative process, starting from given initial guesses, it is important to have good initial guesses to assure convergence.

Permeability was approximated by that obtained from the lowest flow rate observation. This value of permeability together with the following correlation suggested by Heid et al.⁽³⁹⁾ was used for estimating the value of 'b'

$$b = 0.777 k^{-0.39} \quad (41)$$

where b is in international atmospheres and k is in millidarcies. Approximation for the inertial resistance coefficient was obtained using the correlation suggested by Kolada:⁽⁴⁰⁾

$$F_B = \frac{7.56 \times 10^8}{\phi^{1.67} k^{1.12}} \quad (42)$$

where k = millidarcies and ϕ = fractional porosity.

In summary, the following procedure was adopted for parameter estimations by various methods.

- (1) Values of k , b , and F_B were estimated using quasilinearization.
- (2) On the basis of these values the highest flow rate, for which Reynold's number was less than 0.01, was determined.
- (3) The data points, for which the flow rates were less than that determined in (2) above, were plotted on a Klinkenberg plot and values of k and b determined using a linear least squares fit.
- (4) Using the value of ' b ' determined in (3) above, the modified visco-inertial plot was made to yield the values of k and F_B from a linear least squares fit.
- (5) For comparison, the conventional visco-inertial plot was also made.

RESULTS

The physical properties of the samples studied were as shown in Table 1.

Sample 111 was the first to be studied. A Klinkenberg plot for it is presented in Figure 4. Departure from linearity, marking the transition to visco-inertial flow, appears to start at flow rates higher than 0.0937 MSCF/day. This checks with the Reynold's number criterion of $Nr_e = 0.01$. Using the value of 'b' obtained from this plot, the modified visco-inertial plot was made as shown in Figure 5. A conventional visco-inertial plot also appears in the same figure.

Starting with the initial guesses of

$$k = 4.679 \text{ md}$$

$$b = 6.255 \text{ psia}$$

and

$$F_B = 1.679 \times 10^8 \text{ ft}^{-1}$$

quasilinearization was applied using equation (35) and a convergence criterion of $\epsilon = 10^{-4}$. Finally the parameters obtained by means of quasilinearization were used to generate data which were plotted in the form of a conventional visco-inertial plot as shown in Figure 5. A sample calculation for this method of data generation is given in Appendix B.

TABLE 1

PHYSICAL PROPERTIES OF THE SAMPLE

Sample No. Source Description	2		
	111 Imperial Oil Labs. Indiana Limestone	112 Imperial Oil Labs. Indiana Limestone	Sharp Bros. Cut Stone Co. Indiana Limestone
Outer Radius (ft)	0.48835	0.48869	0.49100
Bore Hold Radius (ft)	0.08587	0.08551	0.02125
Thickness (ft)	0.12575	0.12133	0.07933
Pore Volume (cm ³)	373.11	336.49	185.91
Porosity	0.1443	0.1347	0.1095

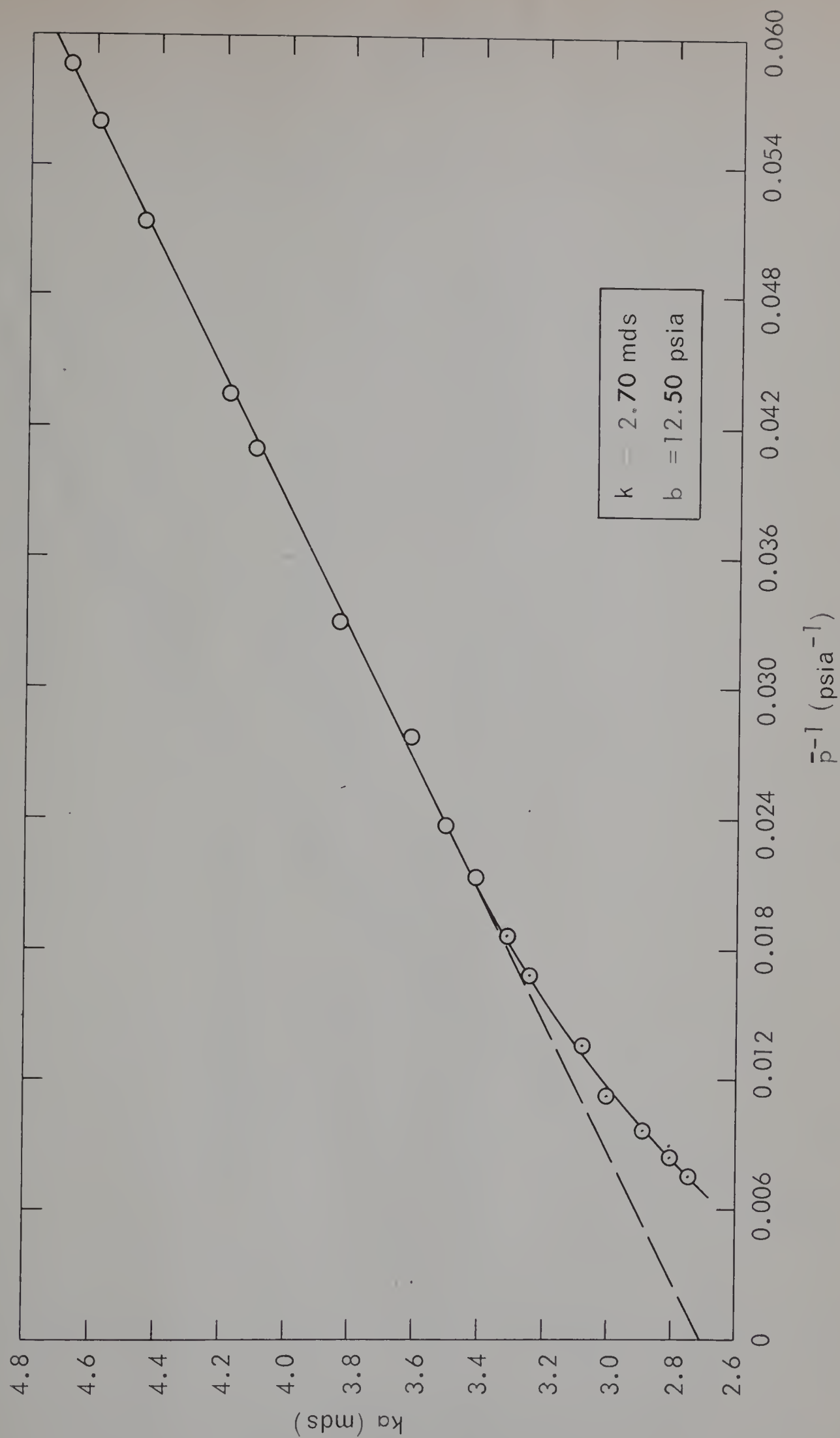


FIGURE 4. KLINKENBERG PLOT SAMPLE 111

N.B. IN THIS AND ALL SUCCEEDING PLOTS A POINT CONTAINING A CENTRAL DOT DESIGNATES A DATA POINT IN THE VISCO-INERTIAL REGION AND A POINT WITHOUT A CENTRAL DOT DESIGNATES A DATA POINT IN THE VISCOUS REGION.

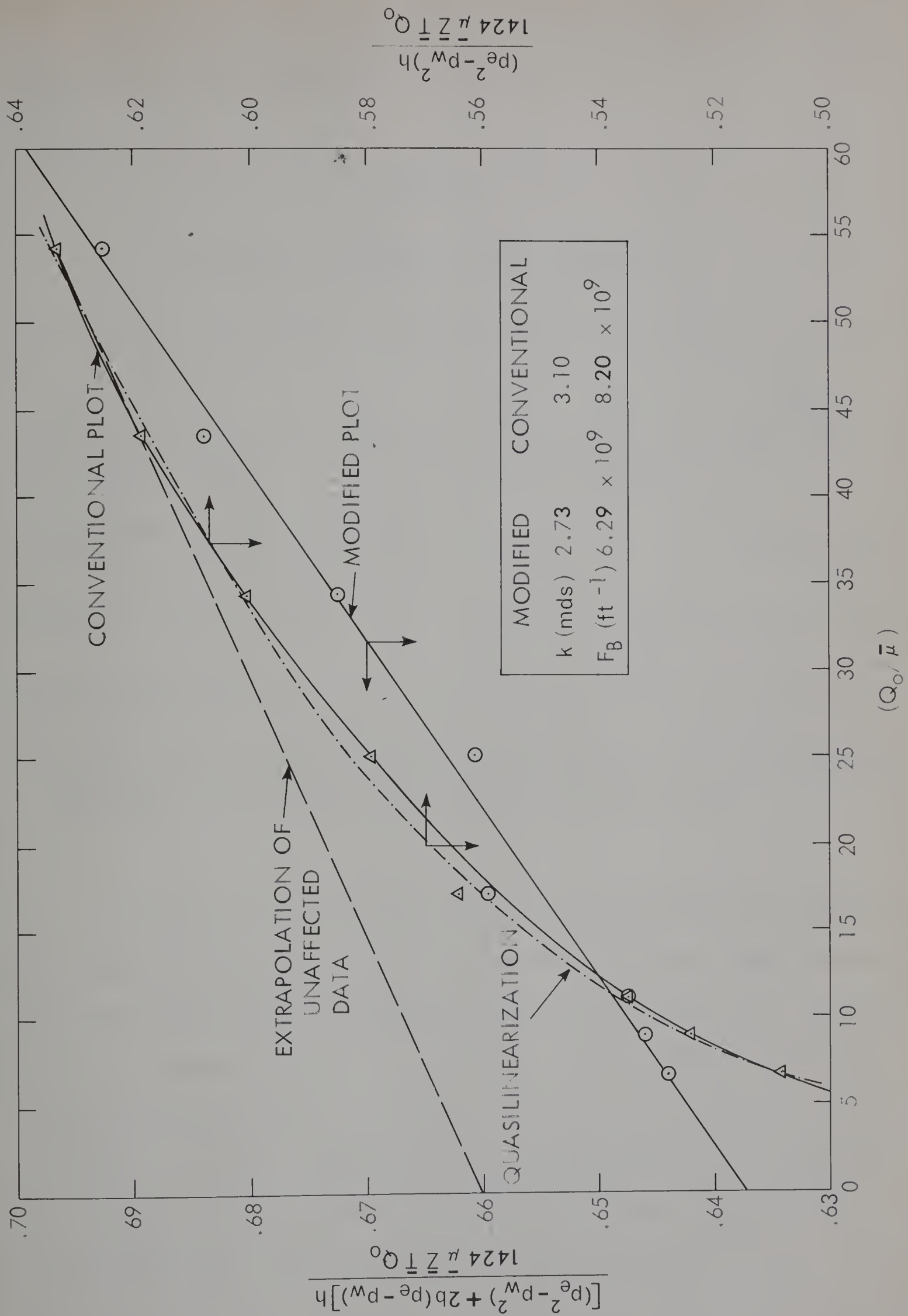


FIGURE 5. VISCO-INERTIAL FLOW PLOTS SAMPLE 111

The parameter estimations obtained graphically and numerically are summarized in Table 2.

TABLE 2

A SUMMARY OF RESULTS FOR SAMPLE 111

Method	k (md)	b (psia)	F_B^{-1} (ft ⁻¹)
Klinkenberg Plot	2.704	12.495	
Conventional Visco-Inertial Plot	3.102		8.198×10^9
Modified Visco-Inertial Plot	2.727	<u>12.495*</u>	6.287×10^9
Quasilinearization	2.719	12.600	6.142×10^9

* in the above table and succeeding tables, an underlined value of b indicates that the value was not determined from that plot but in fact used in its construction.

Adopting the same procedure as in the case of sample 111, of varying the flow rates by holding the downstream pressure at atmospheric and increasing the upstream pressure stepwise, run 1 on sample 112 was recorded. A Klinkenberg plot and visco-inertial plots for this run were constructed as shown in Figures 6 and 7 respectively. From Figure 6 it appears that data points corresponding to flow rates higher than 0.0209 MSCF/day fall outside the viscous region.

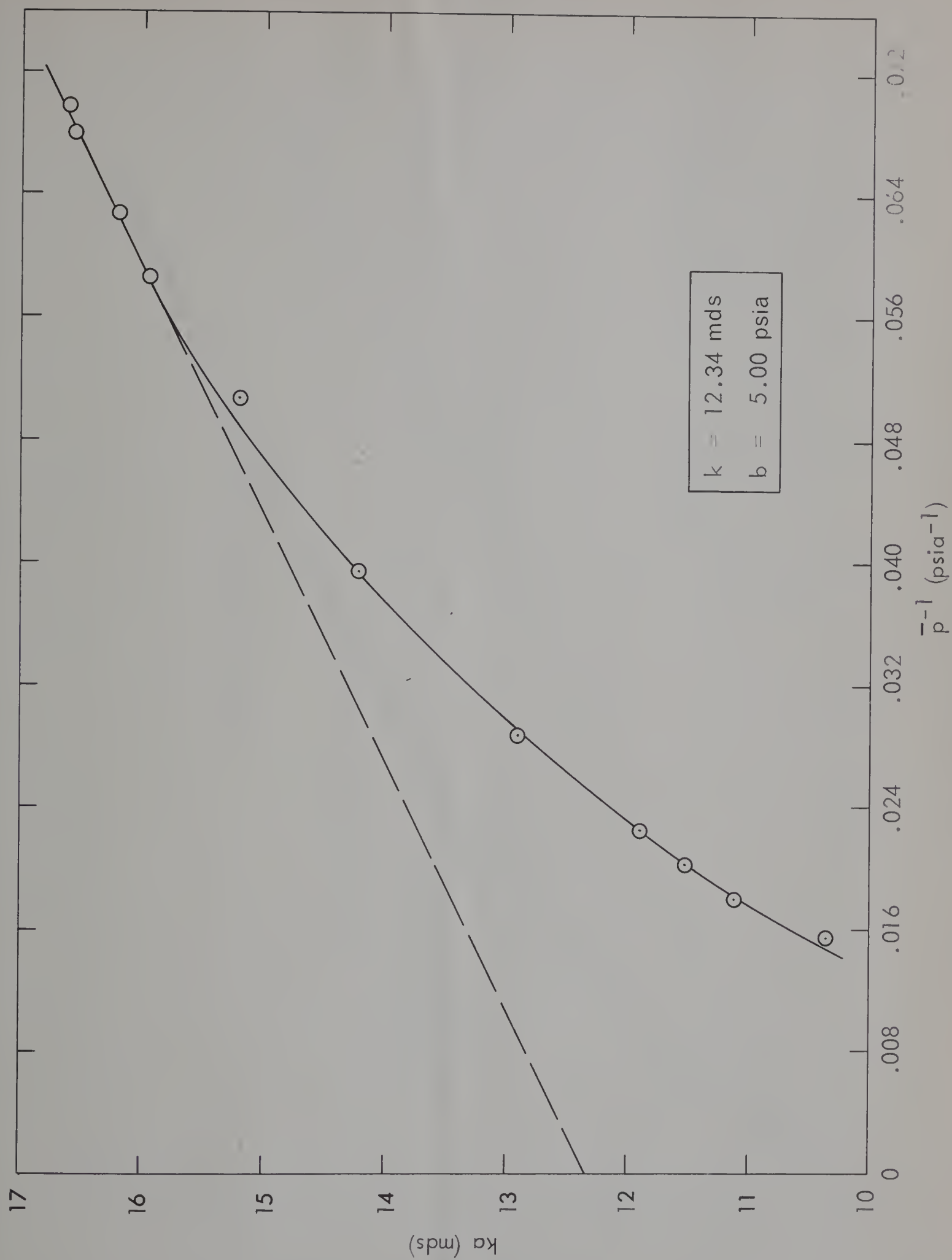


FIGURE 6. KLINKENBERG PLOT SAMPLE 112 RUN 1

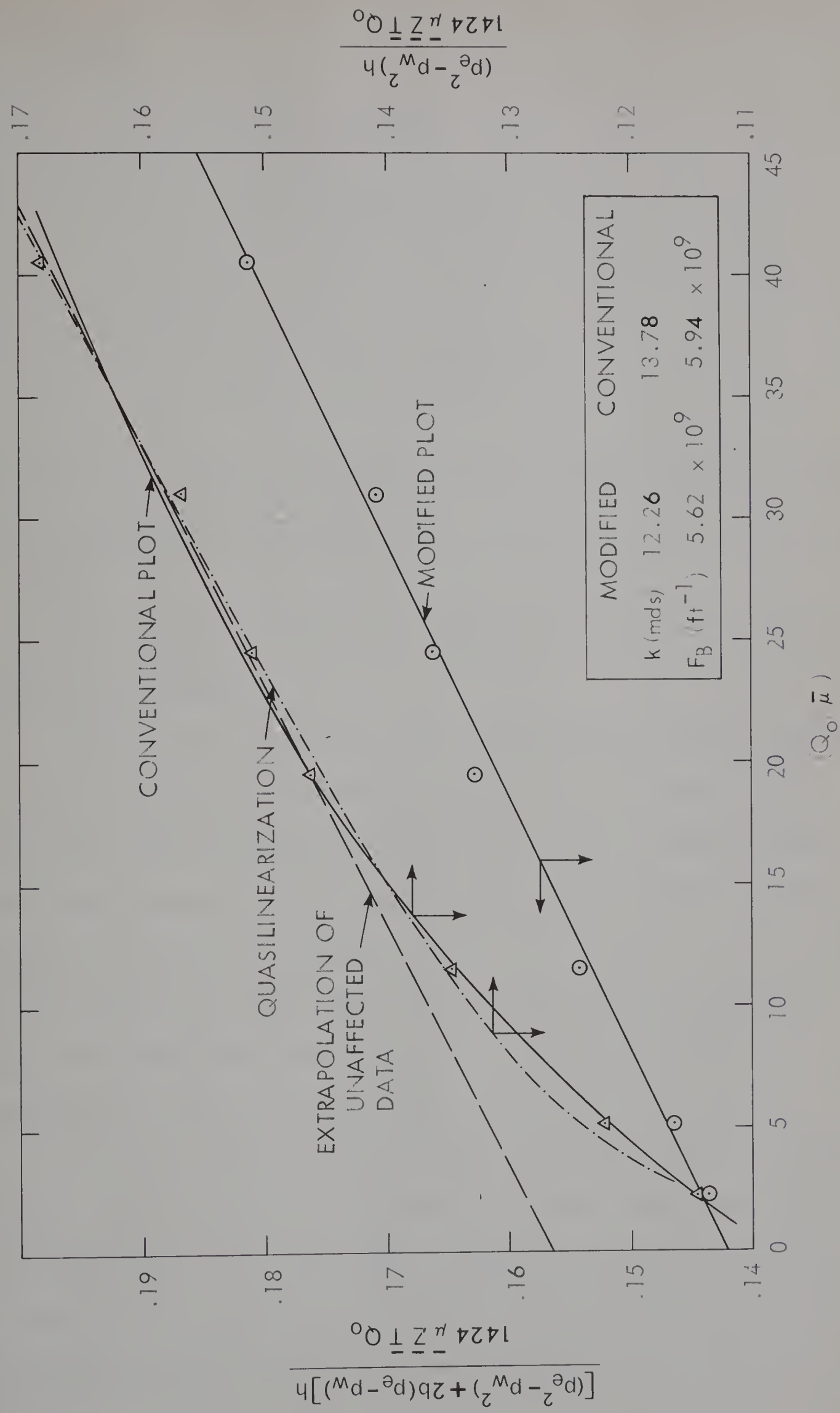


FIGURE 7. VISCO-INERTIAL FLOW PLOTS SAMPLE 112 RUN 1

Using

$$k = 16.617 \text{ md}$$

$$b = 3.815 \text{ psia}$$

and

$$F_B = 9.235 \times 10^8 \text{ ft}^{-1}$$

as initial guesses, five iterations were required for the quasilinearization to attain the convergence criterion of $\epsilon = 10^{-4}$.

The results obtained by these various methods are tabulated in Table 3.

Runs 2 and 3 for the same sample were taken at constant upstream pressures of 100 psig and 80 psig respectively with flow rates being gradually increased by decreasing the downstream pressure. As can be seen from Figure 8, this procedure resulted in peculiar Klinkenberg plots. The nature of these plots is such that they cannot be subjected to conventional interpretation in so far as the prediction of permeability and slippage coefficient is concerned.

Though at first glance these plots may appear to be suspect, the following reasoning suggests that this is in fact what should be expected. When the downstream pressure is constant and upstream pressure is successively increased, $\frac{1}{p}$ decreases while the flow rate increases, resulting in lower

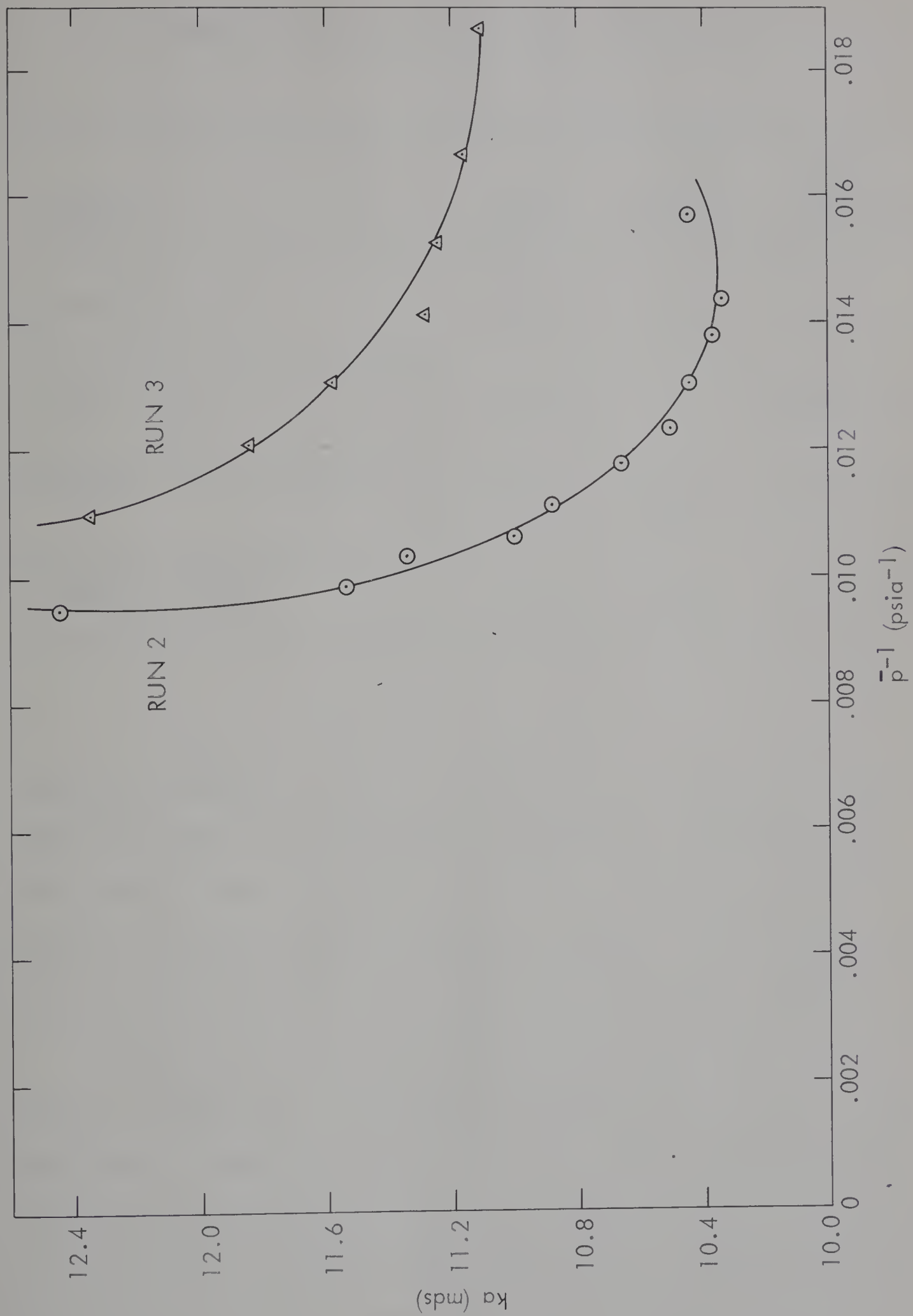


FIGURE 8. KLINKENBERG PLOT SAMPLE 112 RUNS 2 AND 3

TABLE 3

A SUMMARY OF RESULTS FOR RUN 1 SAMPLE 112

Method	k (md)	b (psia)	F_B (ft ⁻¹)
Klinkenberg Plot	12.34	5.00	
Conventional Visco-Inertial Plot	13.78		5.94×10^9
Modified Visco-Inertial Plot	12.26	<u>5.00</u>	5.62×10^9
Quasilinearization of Equation (35)	12.31	4.57	5.55×10^9

apparent permeabilities. However, for a constant upstream pressure when the downstream pressure is decreased, $\frac{1}{p}$ increases. But, since the flow rate also increases, it should result in lower apparent permeabilities. This being precisely what is seen in Figure 8, the given plots are quite in order.

Figure (9) shows the Klinkenberg plots for the three runs on sample 112. As stated earlier, runs 2 and 3 were recorded by imposing back pressure. The difficulty experienced when subjecting these curves to the usual interpretation technique suggests that Sadiq's⁽⁴¹⁾ recommendation concerning the use of back pressure to bring the points up to

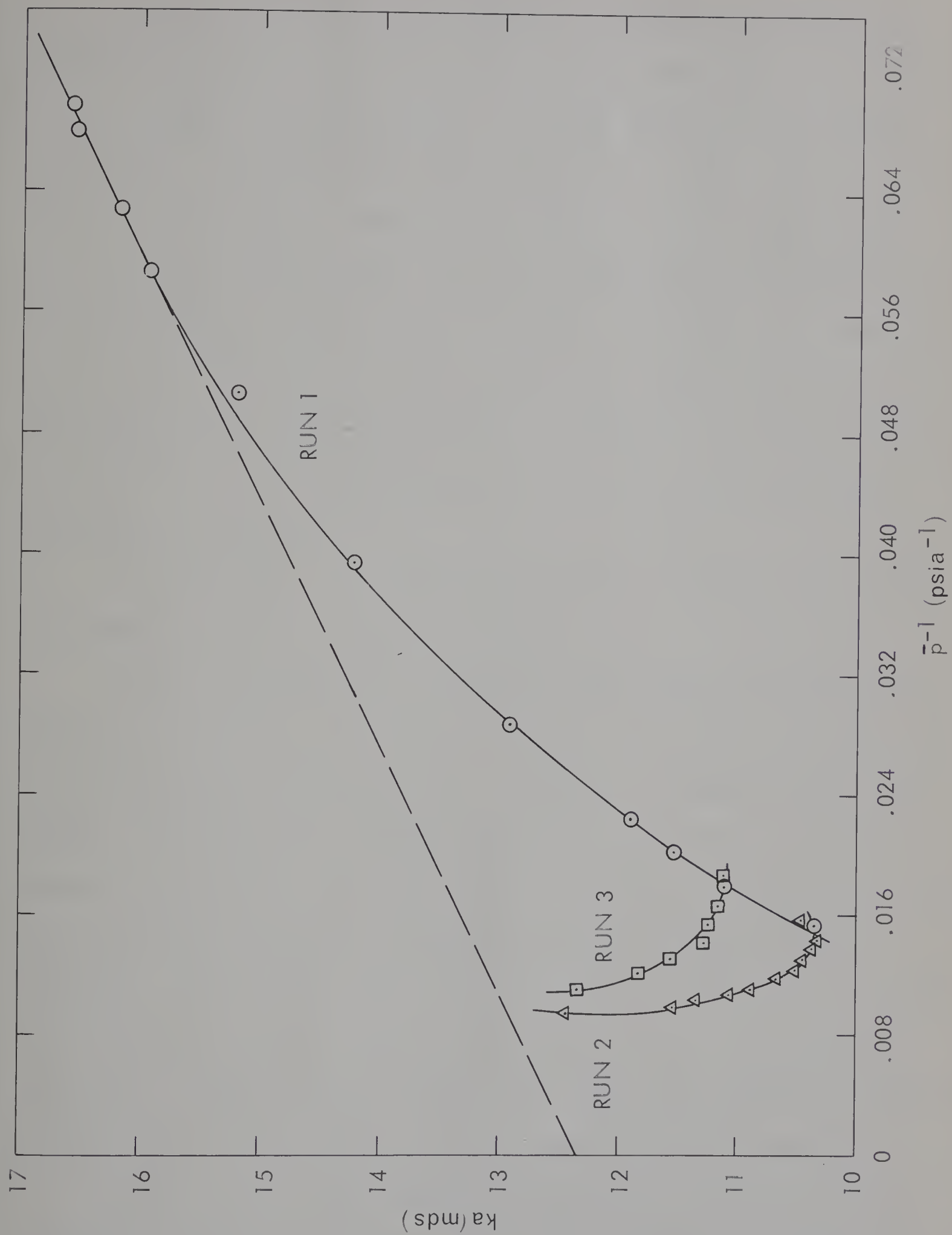


FIGURE 9. KLINKENBERG PLOTS SAMPLE 112 RUNS 1, 2 AND 3

the straight line may not be valid.

Despite the fact that these runs produced anomolous Klinkenberg plots, the resulting visco-inertial plots appear to be normal in every respect, as may be seen from Figures 10 and 11. It should be noted that, as a result of anomolous Klinkenberg plots, parameter estimation by graphical means is not possible in these cases and hence the modified visco-inertial plots cannot be constructed. However, the modified visco-inertial plots shown in these two figures were constructed using the value of 'b' obtained from quasilinearization and are presented merely to show that the resulting behaviour was normal.

A comparison of results obtained for runs 1, 2, and 3 on sample 112 is presented in Table 4.

Run 1 on sample 2, recorded under constant downstream pressure, showed a very unusual behaviour when plotted on a Klinkenberg plot (Figure 12). Contrary to expectation the higher flow rate data points showed an increase in apparent permeability with increasing mean pressure. Though the tentative viscous data gave a permeability of 0.639 md and a slippage coefficient of 7.158 psia, neither the visco-inertial plots (Figure 13) nor quasilinearization gave meaningful values. In fact, both of them gave negative inertial resistance coefficient values which have no known physical meaning.

Run 2 on the same sample, which was recorded with a constant upstream pressure of 200 psig exhibited behaviour

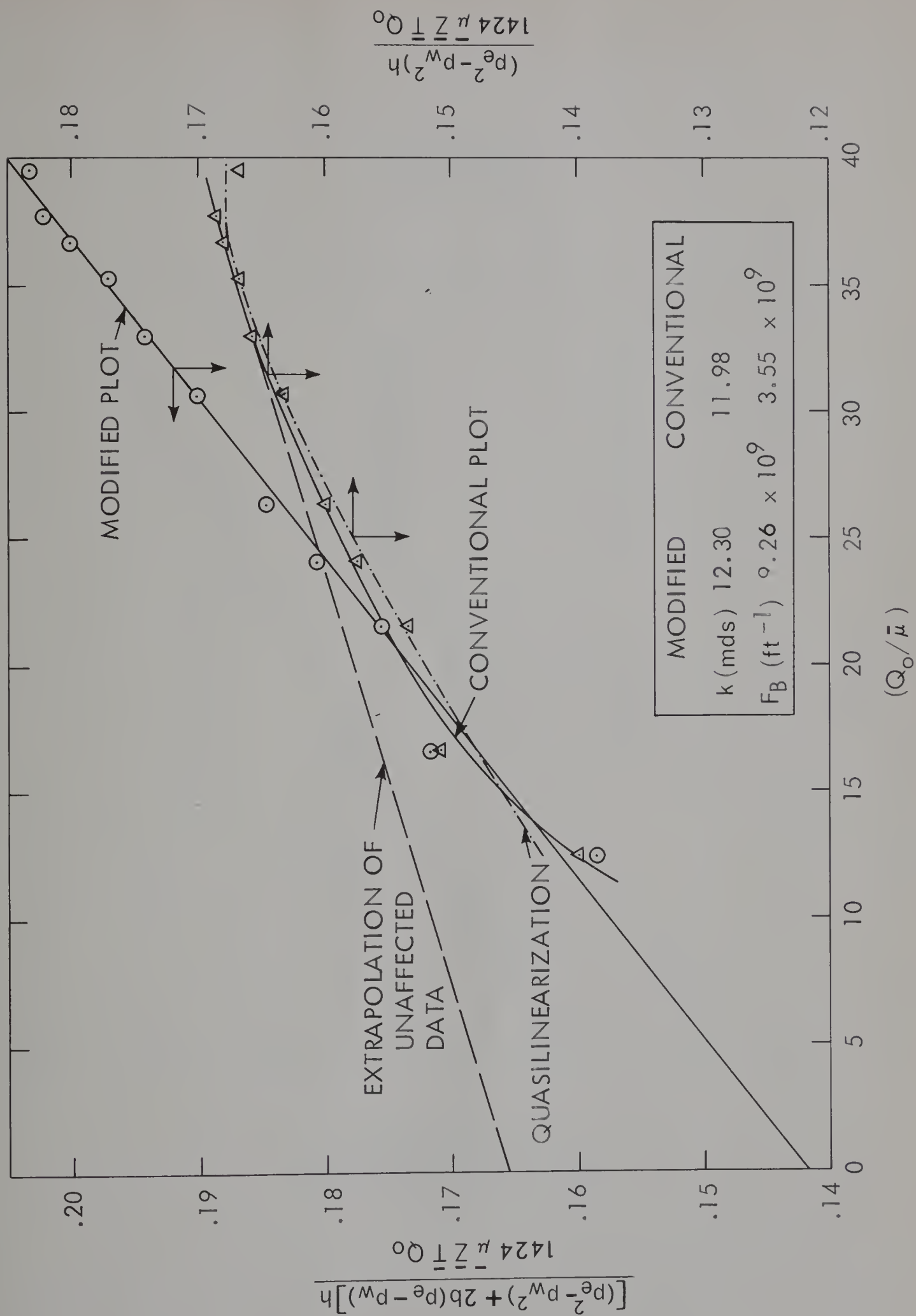


FIGURE 10. VISCO-INERTIAL FLOW PLOTS SAMPLE 112 RUN 2

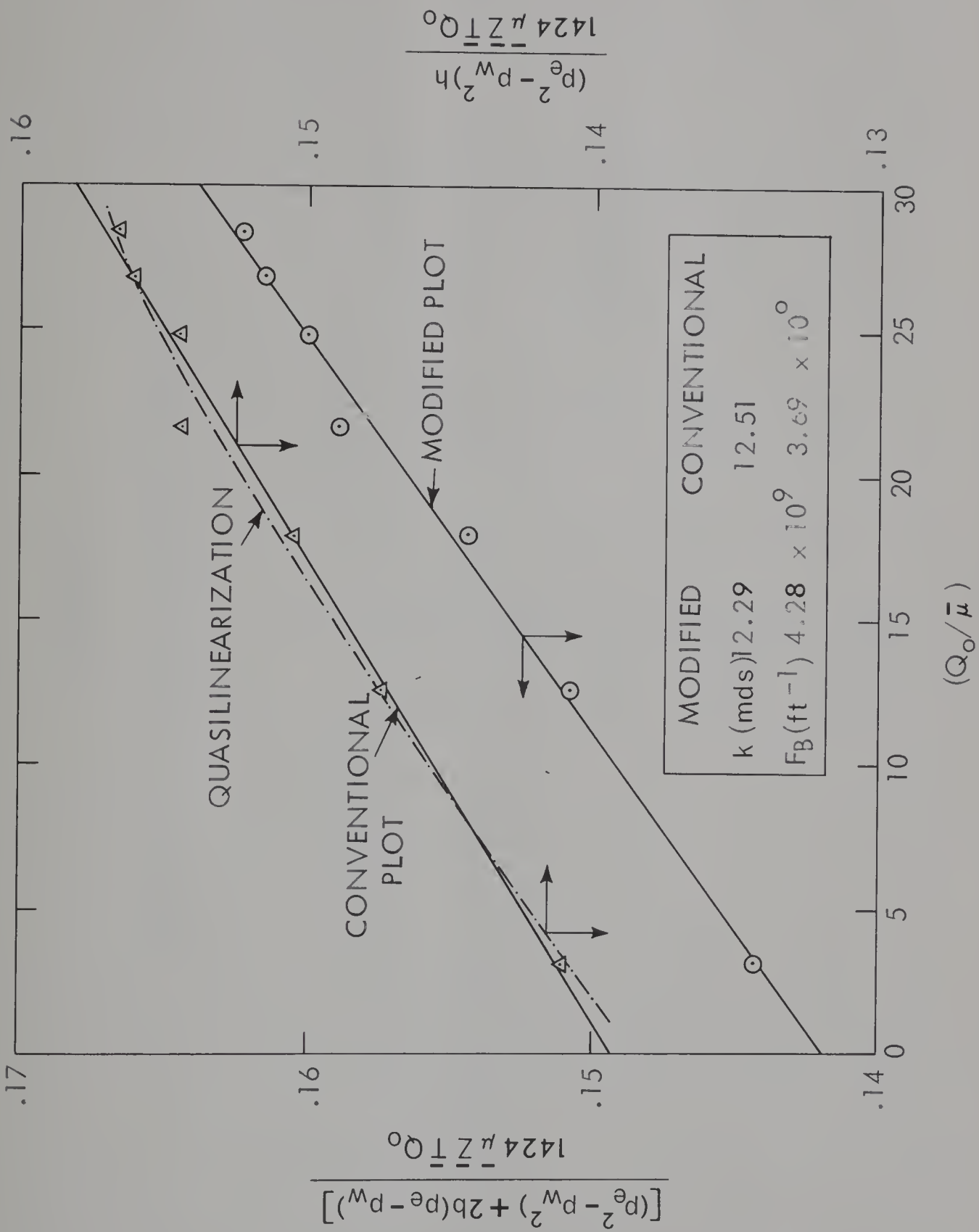


FIGURE 11. VISCO-INERTIAL FLOW PLOTS SAMPLE 112 RUN 3

TABLE 4

RESULTS OF VARIOUS RUNS ON SAMPLE 112

Run No.	Klinkenberg Plot	Conventional Visco-Inertial Plot	Modified Visco-Inertial Plot	Quasilinearization
	k (md) b (psia) F_B^{-1} (ft ⁻¹)	k (md) b (psia) F_B^{-1} (ft ⁻¹)	k (md) b (psia) F_B^{-1} (ft ⁻¹)	k (md) b (psia) F_B^{-1} (ft ⁻¹)
1	12.34 5.00	13.78 5.93 ⁹ x 10 ⁹	12.26 5.00 5.62 ⁹ x 10 ⁹	12.31 4.57 5.55 ⁹ x 10 ⁹
2		11.98 3.55 ⁹ x 10 ⁹	12.30 13.97 9.26 ⁹ x 10 ⁹	12.23 13.97 9.15 ⁹ x 10 ⁹
3		12.51 3.69 ⁹ x 10 ⁹	12.28 2.10 4.28 ⁹ x 10 ⁹	12.29 2.10 4.35 ⁹ x 10 ⁹

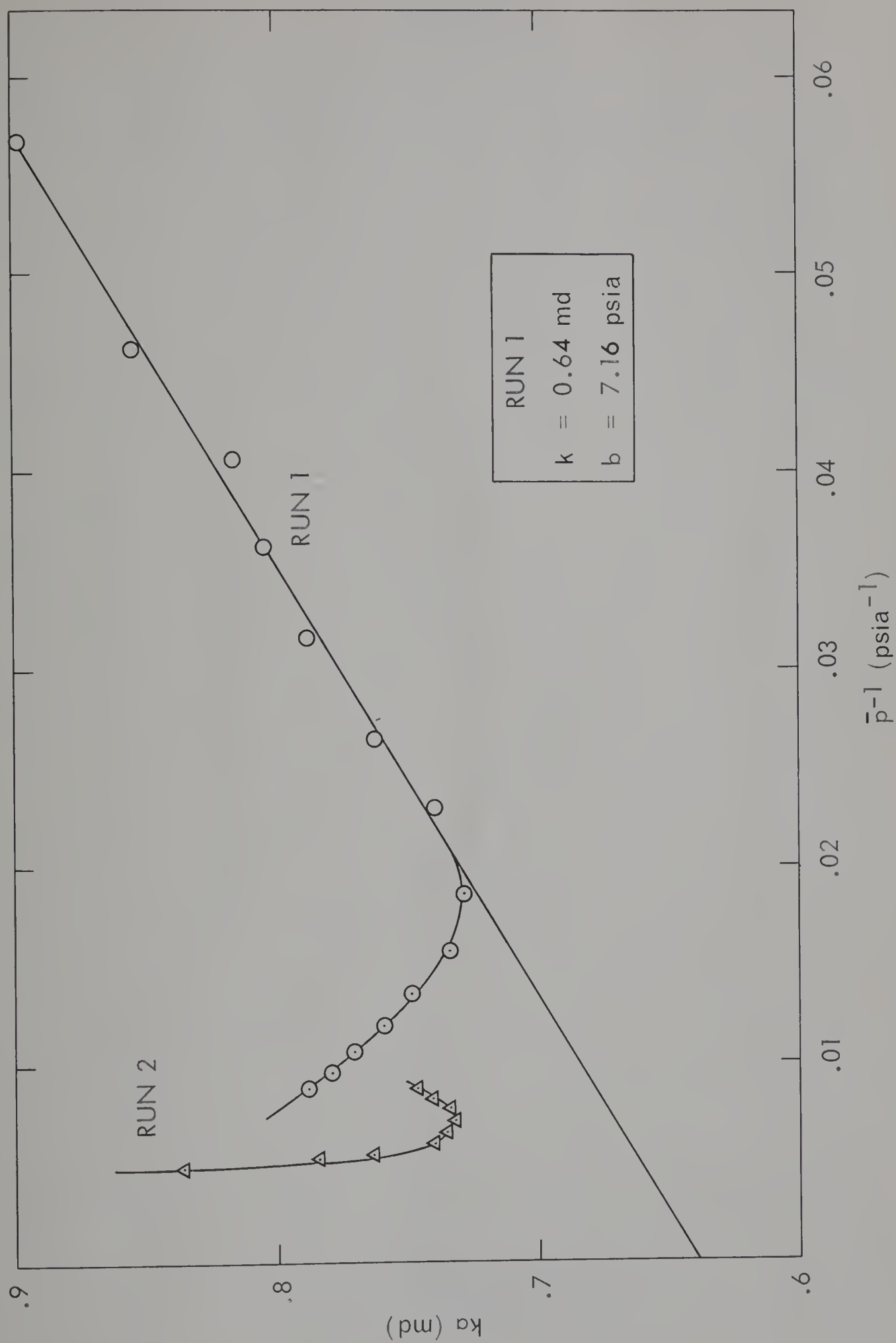


FIGURE 12. KLINKENBERG PLOTS SAMPLE 2 RUNS 1 AND 2

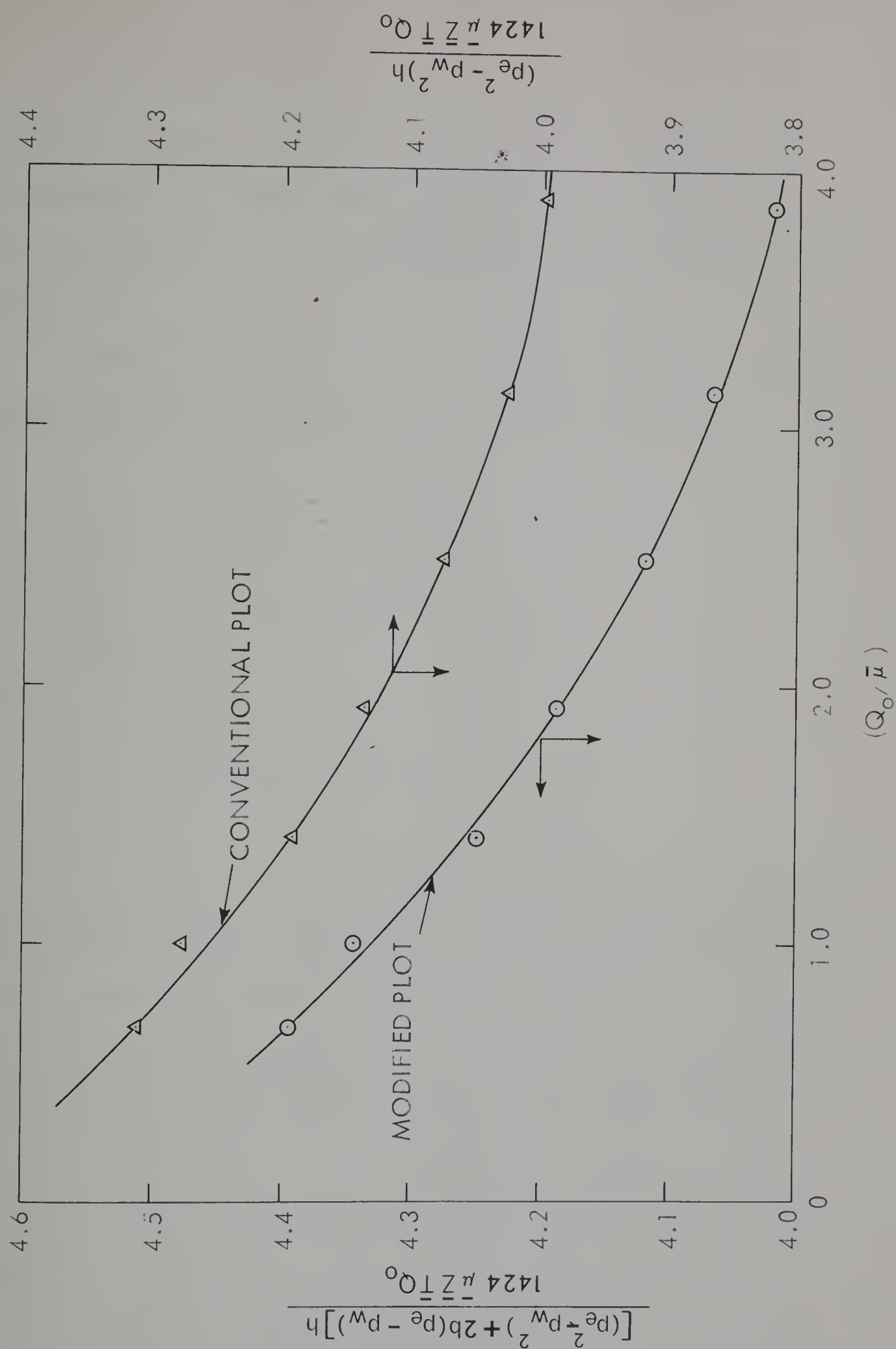


FIGURE 13. VISCO-INERTIAL FLOW PLOTS SAMPLE 2 RUN 1

similar to that observed in runs 2 and 3 on sample 112, as can be seen from Figure 12.

Although the conventional visco-inertial plot for this run was smooth its slope was such that extrapolation was impossible. On the other hand, the modified visco-inertial plot yielded the expected straight line as is evident from Figure 14.

The application of quasilinearization to the data for this run resulted in the following estimates.

$$k = 0.73 \text{ md}$$

$$b = 35.87 \text{ psia}$$

and

$$F_B = 2.71 \times 10^{11} \text{ ft}^{-1}$$

It should be noted that because of the nature of the Klinkenberg plot, parameter estimation by graphical means alone was not possible and that the modified plot in Figure 14 was constructed using the value for 'b' obtained numerically. Furthermore, the modified plots indicated that

$$k = 0.74 \text{ md}$$

and

$$F_B = 2.88 \times 10^{11} \text{ ft}^{-1}$$

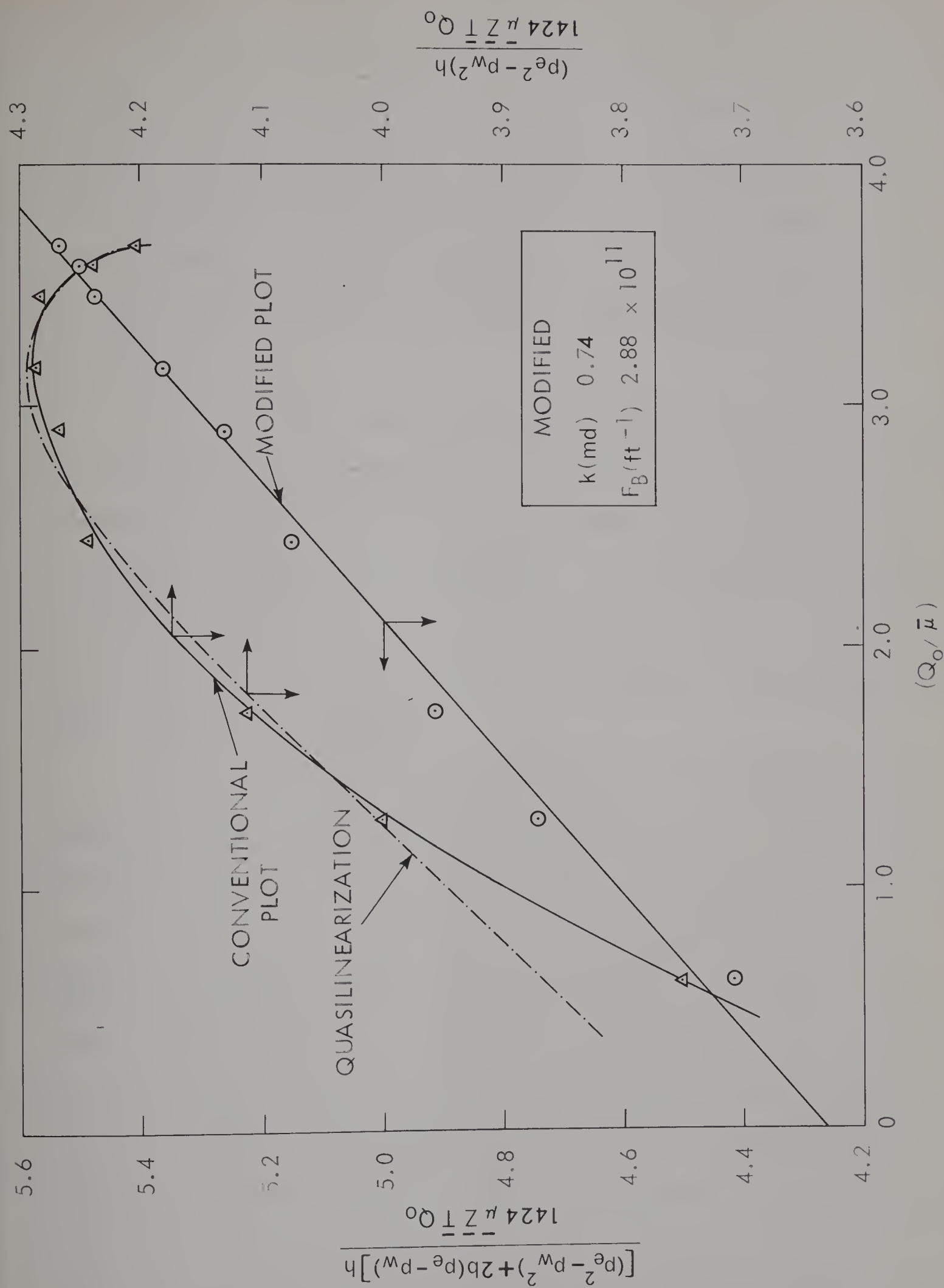


FIGURE 14. VISCO-INERTIAL FLOW PLOTS SAMPLE 2 RUN 2

which merely shows that behaviour was normal.

The abnormal behaviour in Run 1 was thought to be an effect of confining pressure. It was reasoned that increases in mean flowing pressures resulted in decreases in net confining pressures,* which in turn resulted in increases in apparent permeability. The fact that this effect was not very apparent in Run 2 was thought to be due to the difference in test procedure.

In order that this interpretation of the behaviour might be evaluated, run 3 was made. In this case the net confining pressure was held constant at 400 psig and the downstream pressure was held at atmospheric. To further elucidate this behaviour, run 4 was conducted by maintaining a constant net confining pressure of 400 psig and holding the upstream pressure constant at 200 psig.

As may be seen from Figure 15, the Klinkenberg plots for both these runs exhibit normal behaviour. Figure 15A shows the results of run 4 and part of run 3 on an enlarged scale. Visco-inertial plots for these runs are shown in Figures 16 and 17 again indicating normal behaviour. From these plots it was concluded that the reasoning with regards to variations in net confining pressure was essentially correct.

The results of these various parameter estimation techniques as applied to runs 3 and 4 are summarized in Table 5.

* Net confining pressure = Imposed confining pressure
- arithmetic mean flowing pressure.

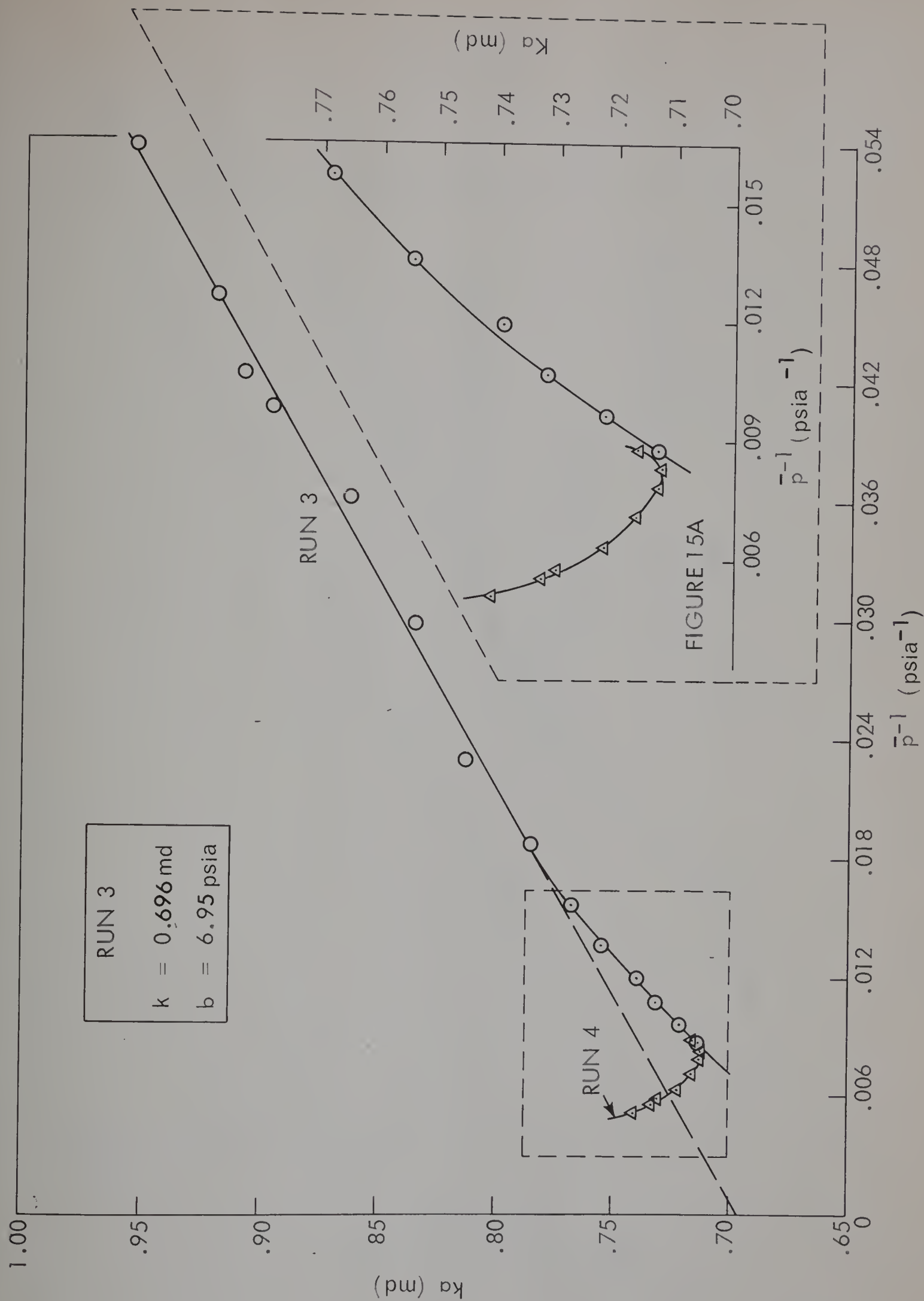


FIGURE 15. KLINKENBERG PLOTS SAMPLE 2 RUNS 3 AND 4

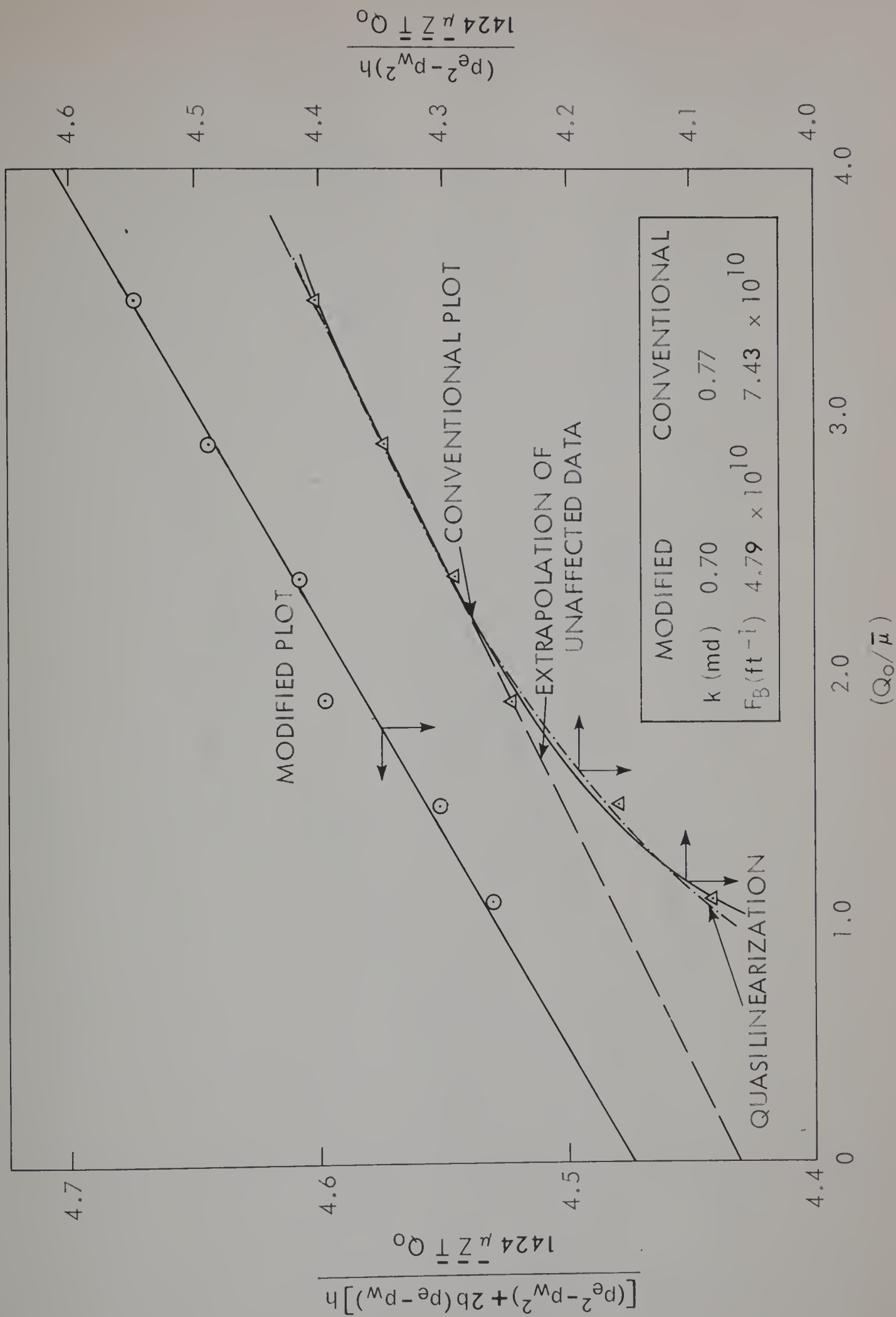


FIGURE 16. VISCO-INERTIAL FLOW PLOTS SAMPLE 2 RUN 3

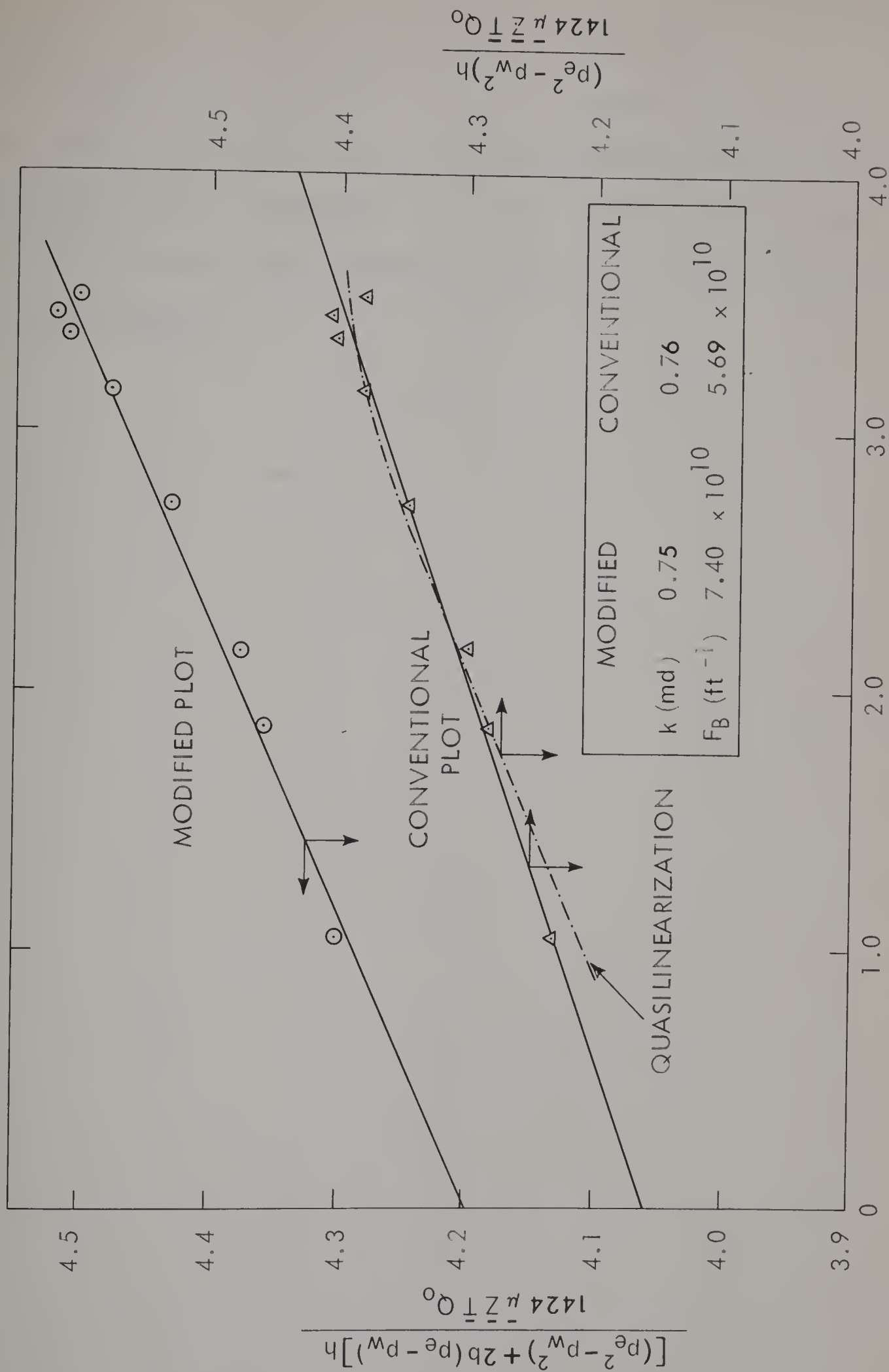


FIGURE 17. VISCO-INERTIAL FLOW PLOTS SAMPLE 2 RUN 4

The observed basic flow data and quasilinearization results for all the runs on the various samples are tabulated in Appendix D. Comparison of the observed function values and those calculated using the parameters obtained from quasilinearization together with the relative percentage error between the two also appear at the end of each table in the same Appendix.

TABLE 5

SUMMARY OF RESULTS FOR RUNS 3 AND 4 ON SAMPLE 2

Run No.	Klinkenberg Plot	Conventional Visco-Inertial Plot	Modified Visco-Inertial Plot	Quasilinearization		
	k (md)	b (psia)	F_B^{-1} (ft ⁻¹)	k (md)	b (psia)	F_B^{-1} (ft ⁻¹)
3	0.696	6.95		0.70	7.02	4.55×10^{10}
4				0.75	3.13	7.88×10^{10}

DISCUSSION OF RESULTS

The modified form of the visco-inertial flow equation is quite amenable to graphical treatment provided the slippage coefficient, b , is known a priori.

Permeability values obtained from the Klinkenberg plot check very well with those predicted from the modified visco-inertial plot and quasilinearization as can be seen from Tables 2, 4 and 5. Also the slippage coefficient values derived from the Klinkenberg plot and quasilinearization show good agreement.

Graphical treatments require a large number of observations since the data have to be split to adequately cover each region. Moreover, as stated earlier, they have another drawback in that the data splitting has to be done on the basis of a Reynold's number, the calculations of which requires the knowledge of k and F_B . Since these parameters happen to be the ones being estimated, a trial and error procedure is required.

On the other hand, the numerical technique of quasilinearization, since it utilizes all of the data simultaneously and is rapid, is preferable if access to a digital computer is available.

This method does have one serious shortcoming in that it uses the data irrespective of their quality. As a result

poor or what may be called "wild" data points cannot be detected in this method, as readily as in the graphical approach as may be seen from Figure 11.

Permeability values obtained from conventional visco-inertial plots, in general, do not correspond to those obtained from Klinkenberg extrapolations. Furthermore, when the slippage coefficient is high it is difficult, if not impossible, to extrapolate the assumed unaffected data. This is well illustrated in Figures 5, 10 and 14.

Even when the slippage coefficient is small, the conventional plot does not yield inertial resistance coefficient values corresponding to those derived from quasilinearization, although permeability values may show good agreement. In this respect, the better agreement in results with other techniques shows the superiority of the modified plotting technique.

One of the most important observations made during this study was that the shape of the Klinkenberg plot is not unique but very much depends on the manner in which a flow experiment is conducted. Figures 8, 9, 12 and 15 bring this out quite clearly.

As shown by the results of Run 1 on Sample 2 (Figure 12) uncontrolled confining pressures could lead to unrealistic data and consequent serious misinterpretations. Hence, the control of confining pressure, at least in certain cases, could very well be an important factor of consideration.

In all of the runs, taken while holding the upstream pressure constant, the observed points were outside the viscous region. This is to be expected since, at higher pressures, even a small pressure drop results in a large flow rate.

Permeability values predicted from the flow experiments conducted at constant upstream pressure compare well with those obtained from the experiment with constant downstream pressure. However, the slippage and inertial resistance coefficients show different values in each case. It is not readily clear whether this is due to some shortcomings in either the experimental or interpretation technique or whether it is a manifestation of some hitherto unknown effect.

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

As a result of this study, the following conclusions were drawn:

- (1) The approximate solution, to the suggested differential equation, appears to describe reasonably well, the radial gas flow through porous media.
- (2) The conventional visco-inertial plot does not seem to be adequate for estimation of permeability and inertial resistance coefficient values.
- (3) The modified visco-inertial plot and the Klinkenberg plot may be used to estimate values of permeability, slippage and inertial resistance coefficients. However, the accuracy of these appears to be somewhat lower than that realized using quasilinearization.
- (4) Gas slippage seems to affect the flow behaviour even at high flow rates. This suggests that the use of conventional visco-inertial plot may result in appreciable errors in parameter estimations.
- (5) The Klinkenberg plot does not have a unique shape but is dependent upon the manner in which the flow experiment has been conducted.
- (6) Since the numerical technique of quasilinearization does not require data splitting, it is more efficient than the graphical method.

- (7) Values of slippage and inertial resistance coefficients obtained from published correlations serve adequately as initial guesses for the quasilinearization method. However, they do not seem to correspond well with the actual values.
- (8) Confining pressure and the manner in which it is applied may influence the results obtained in a laboratory flow experiment.
- (9) The observation of the same permeability values but different slippage coefficient and inertial resistance coefficient values for the same sample under different experimental techniques may have been due to shortcomings in either experimental or interpretation techniques. However, the possibility that this variation was due to some unexamined phenomenon cannot be excluded.

Recommendations

- (1) Flow behavior under various constant downstream pressures should be studied to investigate their effect on the shape of the Klinkenberg plot.
- (2) Steady state flow under both varying upstream and downstream pressures should be investigated since this more closely simulates the conditions encountered in actual gas reservoirs.
- (3) Variations in slippage and inertial resistance coefficients under different experimental techniques should be further

investigated.

- (4) The effect of confining pressure should be studied in greater detail.
- (5) If the gas flow behaviour at constant upstream pressure and very low flow rates is to be studied, more sophisticated pressure regulating and measuring devices should be employed.
- (6) In order to ensure uniform confinement and to avoid breakage of samples, the end faces of samples should be lapped as perfectly as possible.
- (7) Use of thick rubber gaskets for confining the cores should be avoided, especially under higher pressures. Otherwise the stress developed by the flowing rubber may cause the specimen to fail.
- (8) Stress distribution in the rock under confinement should be examined.
- (9) The study should be extended to more competent rock samples at pressures normally encountered at reservoir conditions.
- (10) Experimental verification of the theoretical expression for inertial resistance coefficient as given by equation (13) should be attempted.
- (11) Modification of Crafton's equation so as to account for molecular streaming effects should be attempted.

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APPENDIX A

SALIENT POINTS OF THE TECHNIQUE OF QUASILINEARIZATION

Application of the technique of quasilinearization requires evaluation of the partial derivatives of the functional relationship with respect to the parameters.

Equation (35) on rearrangement can be written as

$$(p_e^2 - p_w^2) = - \left[\frac{1424 \bar{\mu} \bar{Z} \bar{T} Q_0 \ln\left(\frac{r_e}{r_w}\right)}{hk} + 2b(p_e - p_w) \right. \\ \left. + \frac{3.1602 \times 10^{-12} F_B \bar{G} \bar{T} \bar{Z} Q_0 |Q_0| \left(\frac{1}{r_w} - \frac{1}{r_e}\right)}{h^2} \right] \quad (A-1)$$

Now

$$f^j = (p_e^2 - p_w^2)^j \quad (A-2)$$

$$f_{i-1}^j = - \left[\frac{1424 \bar{\mu} \bar{Z} \bar{T} Q_0 \ln\left(\frac{r_e}{r_w}\right)}{hk} + 2b(p_e - p_w) \right. \\ \left. + \frac{3.1602 \times 10^{-12} F_B \bar{G} \bar{T} \bar{Z} Q_0 |Q_0| \left(\frac{1}{r_w} - \frac{1}{r_e}\right)}{h^2} \right]_{i-1}^j \quad (A-3)$$

where j = the j^{th} data point and $j = 1, 2, \dots, N$

i = the i^{th} iteration and $i = 1, 2, \dots, M$

f^j is the observed value of f for the j^{th} data point.

From equation (A-3)

$$\left(\frac{\partial f}{\partial k}\right)_{i-1}^j = \left[\frac{1424 \overline{\mu Z T Q}_0 \ln\left(\frac{r_e}{r_w}\right)}{k_h^2} \right]_{i-1}^j \quad (\text{A-4})$$

$$\left(\frac{\partial f}{\partial b}\right)_{i-1}^j = - [2(p_e - p_w)]_{i-1}^j \quad (\text{A-5})$$

and

$$\left(\frac{\partial f}{\partial F_B}\right)_{i-1}^j = - \left[\frac{3.1602 \times 10^{-12} \overline{G T Z} Q_0 |Q_0| \left(\frac{1}{r_w} - \frac{1}{r_e}\right)}{h^2} \right]_{i-1}^j \quad (\text{A-6})$$

The nomenclature used in the computer program is

K = absolute permeability, k (initial value)

B = slippage coefficient, b (initial value)

FB = inertial resistance coefficient, F_B (initial value)

THIK = h

GRAV = G

EPSLN = ϵ , the convergence criterion

ROQ = Q_0

XT = $\overline{T}$

PE = p_e

PW = p_w

$$VX = \overline{\mu}$$

$$ZX = \overline{Z}$$

$$D(I) = f^j$$

$$DCALC(I) = f_{i-1}^j$$

$$P(I) = \left(\frac{\partial f}{\partial k} \right)$$

$$Q(I) = \left(\frac{\partial f}{\partial b} \right)$$

$$R(I) = \left(\frac{\partial f}{\partial F_B} \right)$$

$$TPSQ = \sum_{j=1}^N P(I)^2$$

$$TPQ = \sum_{j=1}^N P(I) \times Q(I)$$

etc.

Additional nomenclature used in the computer output is

K1 = absolute permeability, k

B4 = slippage coefficient, b

FB1 = inertial resistance coefficient F_B

DCALC(I) = f_M^j = the function value calculated using the parameters given by quasilinearization.

APPENDIX B

SAMPLE CALCULATIONS USING QUASILINEARIZATION RESULTS

Each of the visco-inertial plots includes a curve generated by using the parameters estimated by means of quasilinearization. The procedure for generating such a curve may be illustrated by means of a sample calculation as follows.

Equation (35) which has been derived to describe gas flow through porous media in radial systems may be rewritten as

$$(p_e^2 - p_w^2) + 2b(p_e - p_w) = - \left[\frac{1424 \bar{\mu} \bar{Z} T Q_0 \ln\left(\frac{r_e}{r_w}\right)}{hk} + \frac{3.1602 \times 10^{-12} F_B \bar{G} \bar{T} \bar{Z} Q_0 |Q_0| \left(\frac{1}{r_w} - \frac{1}{r_e}\right)}{h} \right] \quad (B-1)$$

The application of quasilinearization to the data obtained on sample 111, in order to estimate the parameters in the above equation, resulted in the following:

$$k = 2.719 \text{ md}$$

$$b = 12.600 \text{ psia}$$

and

$$F_B = 6.142 \times 10^9 \text{ ft}^{-1}$$

Taking into account the sign convention for Q_0 , equation (B-1) may be rearranged into the following form.

$$-\frac{(p_e^2 - p_w^2)h}{1424 \bar{\mu} \bar{Z} T Q_0} = \left[\frac{\ln\left(\frac{r_e}{r_w}\right)}{k} + \frac{2.2192 \times 10^{-15} F_B G \left(\frac{1}{r_w} - \frac{1}{r_e}\right)}{h} \left(\frac{|Q_0|}{\bar{\mu}}\right) + \frac{2b(p_e - p_w)h}{1424 \bar{\mu} \bar{Z} T Q_0} \right] \quad (B-2)$$

If it is assumed that the parameters obtained by means of quasilinearization are correct, they may be substituted into equation (B-2) in order to predict behaviour under various conditions. For example, given that

$$p_e = 164.52 \text{ psia} \quad p_w = 13.52 \text{ psia}$$

$$Q_0 = 0.4432 \text{ MSCF/day} \quad \bar{T} = 526.511^\circ \text{R}$$

$$\bar{\mu} = 0.01759 \text{ cp} \quad \bar{Z} = 0.9985$$

$$r_e = 0.48835 \text{ ft} \quad r_w = 0.08587 \text{ ft}$$

$$h = 0.12575 \text{ ft}$$

and that for Nitrogen

$$G = 0.9672 \text{ (from NGPSA Engineering Data Book, 1966, page 155).}$$

one may substitute into the right hand side of equation (B-2) so that

$$\frac{\ln\left(\frac{r_e}{r_w}\right)}{k} = 0.63928 \quad (\text{B-3})$$

$$\frac{2.2192 \times 10^{-15} F_B G \left(\frac{1}{r_w} - \frac{1}{r_e} \right) \left(\frac{|Q_0|}{\bar{\mu}} \right)}{h} = 0.02535 \quad (\text{B-4})$$

and

$$- \frac{2b(p_e - p_w)h}{1424 \bar{\mu} Z T Q_0} = - 0.08199 \quad (\text{B-5})$$

and the sum of these three quantities is 0.58264.

Since this quantity is equal to the left hand side of equation (B-2), it along with the appropriate value of $\left(\frac{Q_0}{\bar{\mu}}\right)$ may be used to establish one point of a generated conventional visco-inertial plot. The entire plot may be generated by repeating this procedure for other values of the variables involved.

Ideally, this curve should be identical to the conventional visco-inertial plot and deviations result from experimental error and the fact that quasilinearization fits the data irrespective of their quality.

APPENDIX C

LISTINGS OF COMPUTER PROGRAMS

MAIN

MAIN

```

C      * * * * *
C
C      THIS PROGRAM CALCULATES THE EXPRESSIONS NECESSARY
C      TO MAKE A KLINKENBERG PLOT IF THE VALUE OF K IS
C      READ IN AS EQUAL TO 1 AND A VISCO-INERTIAL PLOT
C      EXPRESSIONS FOR K EQUAL TO 2
C      IF K = 2, THE VALUE OF SLIPPAGE COEFFICIENT B,
C      SHOULD BE READ IN TO GET THE EXPRESSIONS FOR
C      MODIFIED VISCO-INERTIAL PLOT. IN CASE EXPRESSIONS
C      FOR CONVENTIONAL VISCO-INERTIAL PLOT ARE DESIRED
C      B SHOULD BE EQUAL TO ZERO
C
C      * * * * *
C
      REAL KA(50),PM(50),MW,XX(5),ZZ(5),Y(5),XY(5),PLHS(50)
      *,QMU(50)
      READ(5,71)N,K
71  FORMAT(I3,2X,I1)
      GO TO (30,31),K
31  READ(5,32) B
32  FORMAT(F10.5)
30  READ(5,72)RE,RW,THIK
72  FORMAT(3F10.5)
      WRITE(6,300)
300  FORMAT('1',20X,'P(EXT)      P(WELL)      FLOW RATE      '
1' TEMP      VISCOSITY      Z FACTOR',/,21X,
2'(PSIA)      (PSIA)      (MSCF/DAY)      (DEG F)      (CP)',/)
      I = 1
301  READ(5,74)PE,PW,ROQ,T
74  FORMAT(4F10.5)
C
C      * * * * *
C
C      EVALUATION OF THE VISCOSITY AND COMPRESSIBILITY AT
C      THE ARITHMETIC MEAN TEMPERATURE AND PRESSURE OF
C      THE FLOWING GAS
C
C      * * * * *
C
      XT = T+459.7
      AP = (PE+PW)/29.3920
      APS=AP*AP
      APC=APS*AP
      XX(1)=504.
      XX(2)=522.
      XX(3)=540.
      XX(4)=558.
      XX(5)=576.

```


MAIN

... (CONT'D)

```

MW = 28.016
ZZ(1)=.999987E+00-.369988E-03*AP+.208731E-05*APS
1+.723884E-08*APC
ZZ(2)=.999987E+00-.267705E-03*AP+.200599E-05*APS
1+.532217E-08*APC
ZZ(3)=.999995E+00-.182033E-03*AP+.206590E-05*APS
1+.295211E-08*APC
ZZ(4)=.999996E+00-.102998E-03*AP+.189401E-05*APS
1+.246034E-08*APC
ZZ(5)=.999999E+00-.382719E-04*AP+.192274E-05*APS
1+.705037E-09*APC
VX=100.0*(.1778E-03*(1+.8958E-03*(AP-1.))+.612E-06*(AP
*-1.))*2.
1+.3997E-07*(AP-1.))*3.))+.455E-06*(((T-32.)/1.8)-25.))
ZX = 0.0
YX=(XT-XX(1))*(XT-XX(2))*(XT-XX(3))*(XT-XX(4))*(XT
*-XX(5))
DO 410 J=1,5
WY=ZZ(J)*YX/(XT-XX(J))
YDEN=1.
DO 411 M=1,5
IF(J-M) 412,411,412
412 Y(M)=XX(J)-XX(M)
YDEN=YDEN*Y(M)
411 CONTINUE
XY(J)=WY/YDEN
410 ZX=ZX+XY(J)
WRITE(6,302)PE,PW,ROQ,T,VX,ZX
302 FORMAT('0',18X,6(E10.4,1X))
GO TO (600,700),K
600 KA(I) = (1424.0*VX*ZX*XT*ROQ*ALOG(RE/RW))/(((PE+PW)
** (PE-PW))
1*(THIK))
PM(I) = (2.0/(PE+PW))
IF(I-N) 303,304,304
303 I = I+1
GO TO 301
304 WRITE(6,77)
77 FORMAT('///,5X,'APPARENT PERM (M.D.)',5X,'RECIPROCAL
* MEAN PRESS')
DO 90 I=1,N
WRITE(6,78) KA(I),PM(I)
78 FORMAT('0',2(10X,E13.6))
90 CONTINUE
GO TO 800
700 PLHS(I) = (((PE**2)-(PW**2)) + (2.*B*(PE-PW)))*THIK)
*/ (1424.0
1*VX*ZX*XT*ROQ)
QMU(I) = ROQ/VX
IF(I-N) 305,306,306

```


MAIN

... (CONT'D)

```
305 I = I+1
    GO TO 301
306 WRITE(6,80)
    80 FORMAT(///,5X,'LHS OF VISCO INERTIAL PLOT',5X,'Q
        */VISCOSITY')
        DO 100 I = 1,N
        WRITE(6,81)PLHS(I),QMU(I)
    81 FORMAT('0',10X,E13.6,13X,E13.6)
100 CONTINUE
800 CALL EXIT
    END
```


MAIN

MAIN

```

C      * * * * *
C
C      TAKING THE OBSERVED FLOW DATA AS INPUT, THIS PROGRAM
C      GIVES THE ESTIMATES OF K, B AND FB USING THE METHOD
C      OF QUASILINEARIZATION

```

```

C      * * * * *
C
C      THE NOMENCLATURE AND UNITS OF THE INPUT DATA ARE,
C      RE = OUTER RADIUS OF THE ROCK SAMPLE IN FT.
C      RW = BORE HOLE RADIUS OF THE ROCK SAMPLE IN FT.
C      THIK = THICKNESS OF THE ROCK SAMPLE IN FT.
C      GRAV = SPECIFIC GRAVITY OF THE FLOWING GAS
C      EPSLN = CONVERGENCE CRITERION
C      K = INITIAL GUESS FOR PERMEABILITY IN MILLIDARCIES
C      B = INITIAL GUESS FOR SLIPPAGE COEFFICIENT IN PSIA
C      FB = INITIAL GUESS FOR INERTIAL RESISTANCE
C           COEFFICIENT IN 1/FT.
C      PE = PRESSURE AT THE OUTER RADIUS IN PSIA
C      PW = PRESSURE AT THE BORE HOLE IN PSIA
C      ROQ = FLOW RATE IN MSCF/DAY
C      T = MEAN TEMPERATURE OF THE FLOWING GAS IN DEG.F
C      M = NUMBER OF ITERATIONS
C      N = NUMBER OF POINTS IN THE DATA SET
C      COM(J) = APPROPRIATE COMMENT WHICH DESCRIBES
C              THE DATA SET

```

```

C      * * * * *
C
C      * * * * *
C
C      NOTE - IN ORDER TO AVOID CONVERGENCE TROUBLES THIS
C      PROGRAM SHOULD BE RUN IN DOUBLE PRECISION

```

```

C      * * * * *

```

```

      REAL D(50), P(50), R(50), S(50), Q(50), K, KDET
      REAL DCALC(50), ERROR(50), K1(100), MW
      REAL B4(100), FB1(100), XX(5), ZZ(5), Y(5), XY(5)
      REAL C1(50), C2(50), C3(50), COM(20)
      READ(5,1) (COM(J), J=1,20)
1  FORMAT(20A4)
      WRITE(6,2) (COM(J), J=1,20)
2  FORMAT('0',20X,20A4)
      READ(5,71) N
71 FORMAT(I3)

```


MAIN

... (CONT'D)

C SETTING THE INITIAL GUESSES

```

      READ (5,500) K,B,FB
500  FORMAT(2F10.5,E13.5)
      READ(5,72)RE,RW,THIK,GRAV,EPSLN
72  FORMAT(5F10.5)
      WRITE(6,300)
300  FORMAT(///,20X,'P(EXT)      P(WELL)      FLOW RATE      '
1  'TEMP      VISCOSITY      Z FACTOR',/,21X,
2  '(PSIA)      (PSIA)      (MSCF/DAY)      (DEG F)      (CP)',/
  */)
      I = 1
301  READ(5,74)PE,PW,ROQ,T
74  FORMAT(4F10.5)

```

```

C      * * * * *
C
C      EVALUATION OF THE VISCOSITY AND COMPRESSIBILITY AT
C      THE ARITHMETIC MEAN TEMPERATURE AND PRESSURE OF
C      THE FLOWING GAS
C
C      * * * * *

```

```

      XT = T+459.7
      AP = (PE+PW)/29.3920
      APS=AP*AP
      APC=APS*AP
      XX(1)=504.
      XX(2)=522.
      XX(3)=540.
      XX(4)=558.
      XX(5)=576.
      MW=28.016
      ZZ(1)=.999987E+00-.369988E-03*AP+.208731E-05*APS
1+.723884E-08*APC
      ZZ(2)=.999987E+00-.267705E-03*AP+.200599E-05*APS
1+.532217E-08*APC
      ZZ(3)=.999995E+00-.182033E-03*AP+.206590E-05*APS
1+.295211E-08*APC
      ZZ(4)=.999996E+00-.102998E-03*AP+.189401E-05*APS
1+.246034E-08*APC
      ZZ(5)=.999999E+00-.382719E-04*AP+.192274E-05*APS
1+.705037E-09*APC
      VX=100.0*(.1778E-03*(1+.8958E-03*(AP-1.))+.612E-06*(AP
*-1.))*2.
1+.3997E-07*(AP-1.))*3.))+.455E-06*(((T-32.)/1.8)-25.))
      ZX = 0.0
      YX=(XT-XX(1))*(XT-XX(2))*(XT-XX(3))*(XT-XX(4))*(XT
*-XX(5))
      DO 410 J=1,5

```


MAIN

... (CONT'D)

```

      WY=ZZ(J)*YX/(XT-XX(J))
      YDEN=1.
      DO 411 M=1,5
      IF(J-M) 412,411,412
412  Y(M)=XX(J)-XX(M)
      YDEN=YDEN*Y(M)
411  CONTINUE
      XY(J)=WY/YDEN
410  ZX=ZX+XY(J)
      WRITE(6,302)PE,PW,ROQ,T,VX,ZX
302  FORMAT(' ',18X,6(E10.4,1X))
      D(I) = ((PE**2)-PW**2)
      C1(I) = (1424.0*VX*ZX*XT*ROQ*ALOG(RE/RW))
      C2(I) = (PE-PW)*2.0
      C3(I) = (0.31602E-11*GRAV*XT*ZX*ROQ*ROQ*((1./RW)-(1.
*/RE)))
      IF(I-N) 303,304,304
303  I = I+1
      GO TO 301
304  WRITE(6,77)
      77 FORMAT(///,44X,'K',12X,'B',12X,'FB')
      WRITE(6,78)
      78 FORMAT(41X,' (MD) (PSIA) (1/FT)')
      WRITE(6,76) K,B,FB
      76 FORMAT('0',19X,'INITIAL GUESS ',4(2X,E11.4))
      M = 1
26  TPSQ = 0.0
      TPQ = 0.0
      TPR = 0.0
      TQSQ = 0.0
      TQR = 0.0
      TRSQ = 0.0
      TPD = 0.0
      TQD = 0.0
      TRD = 0.0
      TPS = 0.0
      TQS = 0.0
      TRS = 0.0

C      SETTING THE ELEMENTS OF THE MATRIX AND THE
C      KNOWN VECTOR
C
      DO 21 I = 1,N
      DCALC(I) = (C1(I))/(THIK*K)-(B*C2(I))+(C3(I)*FB)/(THIK
**THIK)
      P(I) = -(C1(I))/(K*K*THIK)
      Q(I) = -C2(I)
      R(I) = (C3(I))/(THIK*THIK)
      S(I) = DCALC(I)-K*P(I)-B*Q(I)-FB*R(I)

```


MAIN

... (CONT'D)

```

TPSQ = TPSQ + P(I) * P(I)
TPQ = TPQ + P(I) * Q(I)
TPR = TPR + P(I) * R(I)
TQSQ = TQSQ + Q(I) * Q(I)
TQR = TQR + Q(I) * R(I)
TRSQ = TRSQ + R(I) * R(I)
TPD = TPD + P(I) * D(I)
TQD = TQD + Q(I) * D(I)
TRD = TRD + R(I) * D(I)
TPS = TPS + P(I) * S(I)
TQS = TQS + Q(I) * S(I)
TRS = TRS + R(I) * S(I)

```

21 CONTINUE

B1 = TPD-TPS

B2 = TQD-TQS

B3 = TRD-TRS

C SOLUTION USING CRAMER'S RULE
C

```

DDET = (TPSQ*(TQSQ*TRSQ-TQR*TQR))-(TPQ*(TPQ*TRSQ-TQR
**TPR))+ (TPR*(T
1PQ*TQR-TQSQ*TPR))

```

```

KDET = (TPQ*(TQR*B3-TRSQ*B2))-(TQSQ*(TPR*B3-TRSQ*B1))
**+(TQR*(TPR*B2
1-TQR*B1))

```

```

BDET = (TPSQ*(TQR*B3-TRSQ*B2))-(TPQ*(TPR*B3-TRSQ*B1))
**+(TPR*(TPR*B2
1-TQR*B1))

```

```

FBDET = (TPSQ*(TQSQ*B3-TQR*B2))-(TPQ*(TPQ*B3-TQR*B1))
**+(TPR*(TPQ*B2
1-TQSQ*B1))

```

K1(M) = KDET/DDET

B4(M) = -BDET/DDET

FB1(M) = FBDET/DDET

WRITE(6,106) M,K1(M),B4(M),FB1(M)

106 FORMAT(20X,'ITERATION NO.',I4,3(2X,E11.4))

C A CHECK ON THE CONVERGENCE OF THE SOLUTION
C

A1 = ABS((K1(M)-K)/K)

A2 = ABS((B4(M)-B)/B)

A3 = ABS((FB1(M)-FB)/FB)

IF(A1-EPSLN) 82,82,81

82 IF(A2-EPSLN) 83,83,81

83 IF(A3-EPSLN) 6,6,81

81 K = K1(M)

B = B4(M)

FB = FB1(M)

MAIN

... (CONT'D)

```

      M = M+1
      IF(M-100) 84,84,6
84   GO TO 26
      6 WRITE(6,108)
108  FORMAT('O',////////,22X,'OBS.FUNCTION          CALC.FUNCTION
      *
      1'REL.ERR.(P.CENT)')//)

```

```

C      * * * * *
C
C      CALCULATIONS OF THE RELATIVE PERCENTAGE ERROR
C      BETWEEN THE OBSERVED FUNCTION VALUE AND THAT
C      CALCULATED USING THE PARAMETERS OBTAINED
C      FROM QUASILINEARIZATION
C
C      * * * * *

```

```

      DO 90 I = 1,N
      DCALC(I) = (C1(I))/(THIK*K)-(B*C2(I))+(C3(I)*FB)/(THIK
**THIK)
      ERROR(I) = ABS(((D(I)-DCALC(I))/D(I))*100.0)
      WRITE(6,107) D(I),DCALC(I),ERROR(I)
107  FORMAT(' ',14X,3(4X,F15.5))
      90 CONTINUE
      CALL EXIT
      END

```


APPENDIX D

TABULATED DATA AND RESULTS OF QUASILINEARIZATION

The following tables from Table D-1 to Table D-8 contain the observed flow data. These data are recorded under a total of 9 columns as explained below.

Column No.	Explanation
1	Serial number of the observed data point.
2	Upstream pressure (p_e) in inch of Hg(*) of psig(**).
3	Downstream pressure (p_w) in inch of Hg(*) or psig (**).
4	Time in seconds required to flow 50 cm ³ (*) or 1 ft ³ (**) of gas.
5	Inlet thermocouple (TC_1) reading in millivolts.
6	Outlet thermocouple (TC_2) reading in millivolts.
7	Barometric pressure in inch of Hg.
8	Room temperature in °F.
9	Pressure shown by water manometer (WM) in inch of water.

TABLE D-1

OBSERVED DATA FOR SAMPLE 111

Confining pressure = 800 psig

1	2	3	4	5	6	7	8	9
1	14.00*	0.00*	22.84*	0.695	0.697	27.71	67.0	
2	17.00*	0.00*	18.47*	0.735	0.741	27.87	68.5	
3	14.70*	9.15*	53.24*	0.757	0.759	27.64	70.0	
4	23.50*	14.89*	30.82*	0.759	0.762	27.54	70.0	
5	26.99*	17.20*	26.00*	0.754	0.760	27.54	70.0	
6	33.50**	0.00*	1873.50**	0.757	0.757	27.46	69.5	
7	45.00**	0.00*	1206.02**	0.755	0.755	27.44	69.5	
8	57.50**	0.00*	829.77**	0.761	0.760	27.43	69.5	
9	67.00**	0.00*	658.00**	0.771	0.763	27.43	69.5	
10	80.50**	0.00*	493.50**	0.770	0.761	27.43	69.5	
11	92.00**	0.00*	398.70**	0.755	0.750	27.53	69.5	
12	121.00**	0.00*	257.22**	0.755	0.751	27.53	69.5	
13	151.00**	0.00*	176.11**	0.758	0.751	27.53	69.5	
14	182.00**	0.00*	129.14**	0.759	0.751	27.53	69.5	
15	210.00**	0.00*	101.75**	0.761	0.751	27.53	69.5	
16	239.00**	0.00*	81.56**	0.761	0.751	27.53	69.5	

NB. In this run, WM was not used.

TABLE D-2

OBSERVED DATA FOR SAMPLE 112, RUN 1

Confining pressure = 500 psig

1	2	3	4	5	6	7	8	9
1	4.13*	0.00*	15061.42**	0.785	0.786	27.12	70.0	0.0
2	5.20*	0.00*	20.70*	0.754	0.764	27.36	70.0	0.0
3	10.10*	0.00*	10.20*	0.765	0.770	27.36	70.0	0.0
4	14.80*	0.00*	3716.54**	0.786	0.787	27.35	70.0	0.0
5	25.90*	0.00*	1914.96**	0.806	0.807	27.14	71.0	0.15
6	24.00**	0.00*	843.42**	0.807	0.806	27.14	70.5	0.15
7	43.00**	0.06*	377.95**	0.800	0.800	27.14	70.0	0.60
8	62.00**	0.14*	224.41**	0.801	0.801	27.14	70.0	1.00
9	72.00**	0.20*	179.72**	0.790	0.790	27.35	70.0	1.30
10	84.25**	0.30*	141.73**	0.798	0.798	27.14	70.0	2.40
11	102.0**	0.53*	108.86**	0.800	0.800	27.14	70.0	4.00

TABLE D-3

OBSERVED DATA FOR SAMPLE 112, RUN 2

Confining pressure = 500 psig

1	2	3	4	5	6	7	8	9
1	100.0**	84.0**	392.13**	0.810	0.806	27.69	71.0	0.50
2	100.0**	76.0**	266.14**	0.810	0.806	27.69	71.0	0.90
3	100.0**	67.0**	206.04**	0.809	0.806	27.69	71.0	1.30
4	100.0**	61.0**	184.72**	0.812	0.807	27.69	71.0	1.70
5	100.0**	52.5**	161.42**	0.810	0.805	27.69	71.0	2.10
6	100.0**	42.5**	144.36**	0.810	0.807	27.69	71.0	2.50
7	100.0**	35.0**	135.70**	0.810	0.807	27.69	71.0	2.80
8	100.0**	26.0**	126.97**	0.809	0.806	27.69	71.0	3.10
9	100.0**	18.0**	122.05**	0.810	0.810	27.69	71.0	3.70
10	100.0**	12.0**	119.09**	0.812	0.812	27.69	71.0	3.90
11	100.0**	0.53*	113.78**	0.812	0.813	27.69	71.0	4.10

TABLE D-4

OBSERVED DATA FOR SAMPLE 112, RUN 3

Confining pressure = 500 psig

1	2	3	4	5	6	7	8	9
1	80.0**	75.4**	1427.82**	0.777	0.779	27.60	71.5	0.05
2	80.0**	58.5**	351.71**	0.786	0.785	27.60	71.5	0.50
3	80.0**	46.0**	246.72**	0.796	0.792	27.60	71.5	1.00
4	80.0**	34.5**	204.72**	0.802	0.798	27.60	71.5	1.30
5	80.0**	23.0**	178.48**	0.807	0.801	27.57	71.5	1.60
6	80.0**	13.0**	165.83**	0.809	0.800	27.57	71.5	1.90
7	80.0**	0.10**	156.69**	0.814	0.804	27.57	71.5	2.10

TABLE D-5

OBSERVED DATA FOR SAMPLE 2, RUN 1

Confining pressure = 500 psig

1	2	3	4	5	6	7	8	9
1	200.0**	0.0*	1154.86**	0.800	0.796	27.67	68.00	0.15
2	180.0**	0.0*	1422.36**	0.804	0.799	27.67	68.00	0.15
3	160.0**	0.0*	1792.13**	0.806	0.801	27.67	68.00	0.10
4	140.0**	0.0*	2326.51**	0.808	0.803	27.67	68.25	0.05
5	120.0**	0.0*	3120.21**	0.805	0.805	27.67	68.50	0.03
6	100.0**	0.0*	4409.45**	0.805	0.805	27.65	68.50	0.0
7	80.0**	0.0*	6582.68**	0.795	0.795	27.65	69.00	0.0
8	60.0**	0.0*	10582.68**	0.804	0.800	27.65	69.00	0.0
9	48.0**	0.0*	26.40*	0.800	0.798	27.66	68.6	0.0
10	36.0**	0.0*	40.50*	0.796	0.795	27.66	68.6	0.0
11	28.0**	0.0*	58.35*	0.793	0.791	27.66	68.6	0.0
12	22.0**	0.0*	82.2*	0.795	0.794	27.66	68.6	0.0
13	16.0**	0.0*	122.4*	0.790	0.790	27.64	70.0	0.0
14	8.0**	0.0*	286.95*	0.790	0.790	27.66	68.6	0.0

TABLE D-6

OBSERVED DATA FOR SAMPLE 2, RUN 2

Confining pressure = 500 psig

1	2	3	4	5	6	7	8	9
1	200.0**	183.0**	12.60*	0.833	0.825	27.69	70.0	0.0
2	200.0**	160.0**	3425.20**	0.840	0.830	27.69	70.0	0.0
3	200.0**	142.0**	2543.31**	0.835	0.830	27.69	70.0	0.08
4	200.0**	108.0**	1815.22**	0.800	0.802	27.69	69.2	0.10
5	200.0**	80.6**	1530.71**	0.795	0.800	27.69	69.0	0.10
6	200.0**	56.0**	1385.83**	0.792	0.798	27.69	68.5	0.15
7	200.0**	30.0**	1289.76**	0.790	0.795	27.69	68.5	0.15
8	200.0**	14.0**	1244.09**	0.790	0.795	27.69	68.3	0.15
9	200.0**	0.0*	1217.85**	0.790	0.794	27.69	68.3	0.15

TABLE D-7

OBSERVED DATA FOR SAMPLE 2, RUN 3

Net Confining pressure = 400 psig

1	2	3	4	5	6	7	8	9
1	10.0**	0.0*	204.90*	0.845	0.847	27.71	71.50	0.0
2	16.0**	0.0*	114.60*	0.854	0.852	27.73	71.60	0.0
3	20.0**	0.0*	84.90*	0.834	0.831	27.73	70.50	0.0
4	22.0**	0.0*	75.00*	0.840	0.841	27.71	71.30	0.0
5	28.0**	0.0*	54.60*	0.835	0.831	27.74	70.50	0.0
6	40.0**	0.0*	32.40*	0.836	0.840	27.71	71.30	0.0
7	60.0**	0.0*	17.1*	0.830	0.831	27.71	71.00	0.0
8	80.0**	0.0*	6137.54**	0.812	0.812	27.80	69.70	0.02
9	100.0**	0.0*	4230.37**	0.813	0.815	27.80	69.70	0.03
10	120.0**	0.0*	3105.38**	0.817	0.817	27.80	69.70	0.05
11	140.0**	0.0*	2393.70**	0.817	0.819	27.80	69.70	0.05
12	160.0**	0.0*	1891.86**	0.823	0.821	27.80	69.90	0.07
13	180.0**	0.0*	1541.19**	0.823	0.823	27.80	70.00	0.10
14	200.0**	0.0*	1281.89**	0.825	0.824	27.80	70.00	0.10

TABLE D-8

OBSERVED DATA FOR SAMPLE 2, RUN 4

Net Confining pressure = 400 psig

1	2	3	4	5	6	7	8	9
1	200.0**	0.0*	1259.84**	0.787	0.785	27.48	68.0	0.15
2	200.0**	14.38**	1286.17**	0.796	0.791	27.48	68.0	0.15
3	200.0**	28.63**	1314.96**	0.796	0.794	27.48	68.0	0.10
4	200.0**	52.88**	1401.58**	0.831	0.821	27.64	69.6	0.0
5	200.0**	87.75**	1623.62**	0.836	0.826	27.64	69.6	0.0
6	200.0**	120.0**	2047.24**	0.841	0.831	27.64	69.5	0.0
7	200.0**	133.50**	2363.70**	0.873	0.876	27.68	73.2	0.0
8	200.0**	165.95**	4183.83**	0.880	0.882	27.65	74.0	0.0

TABLE D - 9

PROCESSED DATA AND QUASILINEARIZATION RESULTS SAMPLE 111

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/DAY)	TEMP (DEG F)	VISCOSITY (CP)	Z FACTOR
0.2048E 02	0.1361E 02	0.6100E-02	0.6389E 02	0.1745E-01	0.9996E 00
0.2203E 02	0.1368E 02	0.7560E-02	0.6577E 02	0.1749E-01	0.9996E 00
0.2079E 02	0.1806E 02	0.2590E-02	0.6668E 02	0.1752E-01	0.9996E 00
0.2506E 02	0.2083E 02	0.4460E-02	0.6678E 02	0.1753E-01	0.9996E 00
0.2678E 02	0.2197E 02	0.5290E-02	0.6663E 02	0.1752E-01	0.9995E 00
0.4698E 02	0.1348E 02	0.4284E-01	0.6661E 02	0.1753E-01	0.9994E 00
0.5847E 02	0.1347E 02	0.6450E-01	0.6653E 02	0.1753E-01	0.9993E 00
0.7097E 02	0.1347E 02	0.9372E-01	0.6679E 02	0.1755E-01	0.9993E 00
0.8047E 02	0.1347E 02	0.1181E 00	0.6709E 02	0.1756E-01	0.9992E 00
0.9397E 02	0.1347E 02	0.1575E 00	0.6702E 02	0.1757E-01	0.9991E 00
0.1055E 03	0.1352E 02	0.1957E 00	0.6643E 02	0.1756E-01	0.9990E 00
0.1345E 03	0.1352E 02	0.3034E 00	0.6644E 02	0.1757E-01	0.9987E 00
0.1645E 03	0.1352E 02	0.4431E 00	0.6651E 02	0.1759E-01	0.9985E 00
0.1955E 03	0.1352E 02	0.6043E 00	0.6653E 02	0.1761E-01	0.9983E 00
0.2235E 03	0.1352E 02	0.7670E 00	0.6659E 02	0.1763E-01	0.9981E 00
0.2525E 03	0.1352E 02	0.9569E 00	0.6657E 02	0.1764E-01	0.9979E 00

	K (MD)	B (PSIA)	FB (1/FT)
INITIAL GUESS	0.4679E 01	0.6255E 01	0.1679E 09
ITERATION NO. 1	0.1307E 01	0.1259E 02	0.6145E 10
ITERATION NO. 2	0.1986E 01	0.1259E 02	0.6144E 10
ITERATION NO. 3	0.2522E 01	0.1259E 02	0.6145E 10
ITERATION NO. 4	0.2705E 01	0.1259E 02	0.6145E 10
ITERATION NO. 5	0.2719E 01	0.1259E 02	0.6145E 10
ITERATION NO. 6	0.2719E 01	0.1259E 02	0.6145E 10

. . . CONTD

TABLE D - 9

OBS.FUNCTION	CALC.FUNCTION	REL.ERR.(P.CENT)
0.23445E 03	0.23022E 03	0.18050E 01
0.29831E 03	0.29291E 03	0.18069E 01
0.10594E 03	0.10420E 03	0.16441E 01
0.19414E 03	0.19136E 03	0.14312E 01
0.23444E 03	0.23206E 03	0.10169E 01
0.20258E 04	0.20266E 04	0.37512E-01
0.32379E 04	0.31966E 04	0.12746E 01
0.48555E 04	0.48672E 04	0.24076E 00
0.62943E 04	0.63033E 04	0.14273E 00
0.86493E 04	0.86654E 04	0.18692E 00
0.10951E 05	0.10988E 05	0.33058E 00
0.17913E 05	0.17787E 05	0.70353E 00
0.26884E 05	0.27025E 05	0.52448E 00
0.38045E 05	0.38076E 05	0.79779E-01
0.49779E 05	0.49645E 05	0.26917E 00
0.63584E 05	0.63642E 05	0.91054E-01

TABLE D - 10

PROCESSED DATA AND QUASILINEARIZATION RESULTS SAMPLE 112 RUN 1

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/DAY)	TEMP (DEG F)	VISCOSITY (CP)	Z FACTOR
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0.1534E 02	0.1332E 02	0.5100E-02	0.6793E 02	0.1755E-01	0.9997E 00
0.1599E 02	0.1343E 02	0.6610E-02	0.6653E 02	0.1751E-01	0.9997E 00
0.1839E 02	0.1343E 02	0.1355E-01	0.6711E 02	0.1753E-01	0.9997E 00
0.2070E 02	0.1343E 02	0.2090E-01	0.6797E 02	0.1755E-01	0.9997E 00
0.2605E 02	0.1332E 02	0.4010E-01	0.6888E 02	0.1758E-01	0.9996E 00
0.3732E 02	0.1332E 02	0.9113E-01	0.6888E 02	0.1758E-01	0.9995E 00
0.5632E 02	0.1335E 02	0.2037E 00	0.6859E 02	0.1758E-01	0.9994E 00
0.7532E 02	0.1339E 02	0.3440E 00	0.6863E 02	0.1760E-01	0.9992E 00
0.8543E 02	0.1353E 02	0.4327E 00	0.6813E 02	0.1759E-01	0.9992E 00
0.9757E 02	0.1347E 02	0.5461E 00	0.6850E 02	0.1760E-01	0.9991E 00
0.1153E 03	0.1359E 02	0.7141E 00	0.6859E 02	0.1762E-01	0.9989E 00

	K (MD)	B (PSIA)	FB (1/FT)
INITIAL GUESS	0.1661E 02	0.3815E 01	0.9235E 09
ITERATION NO. 1	0.1081E 02	0.4562E 01	0.5547E 10
ITERATION NO. 2	0.1213E 02	0.4562E 01	0.5547E 10
ITERATION NO. 3	0.1231E 02	0.4562E 01	0.5547E 10
ITERATION NO. 4	0.1231E 02	0.4562E 01	0.5547E 10
ITERATION NO. 5	0.1231E 02	0.4562E 01	0.5547E 10

. . . CONTD

TABLE D - 10

OBS. FUNCTION	CALC. FUNCTION	REL. ERR. (P. CENT)
0.58153E 02	0.60048E 02	0.32574E 01
0.75164E 02	0.78095E 02	0.38999E 01
0.15793E 03	0.16356E 03	0.35685E 01
0.24813E 03	0.25762E 03	0.38264E 01
0.50096E 03	0.51196E 03	0.21969E 01
0.12158E 04	0.12359E 04	0.16538E 01
0.29945E 04	0.29937E 04	0.28469E-01
0.54950E 04	0.54390E 04	0.10189E 01
0.71157E 04	0.71089E 04	0.94417E-01
0.93402E 04	0.94097E 04	0.74453E 00
0.13116E 05	0.13090E 05	0.19511E 00

TABLE D - 11

PROCESSED DATA AND QUASILINEARIZATION RESULTS SAMPLE 112 RUN 2

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/DAY)	TEMP (DEG F)	VISCOSITY (CP)	Z FACTOR
0.1136E 03	0.9760E 02	0.2203E 00	0.6895E 02	0.1767E-01	0.9984E 00
0.1136E 03	0.8960E 02	0.2949E 00	0.6895E 02	0.1767E-01	0.9984E 00
0.1136E 03	0.8060E 02	0.3814E 00	0.6893E 02	0.1766E-01	0.9985E 00
0.1136E 03	0.7460E 02	0.4258E 00	0.6902E 02	0.1766E-01	0.9985E 00
0.1136E 03	0.6610E 02	0.4878E 00	0.6893E 02	0.1765E-01	0.9986E 00
0.1136E 03	0.5610E 02	0.5461E 00	0.6897E 02	0.1765E-01	0.9987E 00
0.1136E 03	0.4860E 02	0.5814E 00	0.6897E 02	0.1764E-01	0.9987E 00
0.1136E 03	0.3960E 02	0.6218E 00	0.6893E 02	0.1764E-01	0.9988E 00
0.1136E 03	0.3160E 02	0.6479E 00	0.6904E 02	0.1764E-01	0.9988E 00
0.1136E 03	0.2560E 02	0.6643E 00	0.6913E 02	0.1764E-01	0.9989E 00
0.1136E 03	0.1386E 02	0.6971E 00	0.6915E 02	0.1763E-01	0.9990E 00

	K (MD)	B (PSIA)	FB (1/FT)
INITIAL GUESS	0.1661E 02	0.3815E 01	0.9235E 09
ITERATION NO. 1	0.1065E 02	0.1398E 02	0.9151E 10
ITERATION NO. 2	0.1202E 02	0.1398E 02	0.9151E 10
ITERATION NO. 3	0.1222E 02	0.1398E 02	0.9151E 10
ITERATION NO. 4	0.1223E 02	0.1398E 02	0.9151E 10
ITERATION NO. 5	0.1223E 02	0.1398E 02	0.9151E 10

. . . CONTD

TABLE D - 11

OBS.FUNCTION	CALC.FUNCTION	REL.ERR.(P.CENT)
0.33792E 04	0.34606E 04	0.24097E 01
0.48768E 04	0.47728E 04	0.21322E 01
0.64086E 04	0.64340E 04	0.39732E 00
0.73398E 04	0.73083E 04	0.42924E 00
0.85357E 04	0.85822E 04	0.54488E 00
0.97577E 04	0.97929E 04	0.36086E 00
0.10543E 05	0.10517E 05	0.23768E 00
0.11336E 05	0.11364E 05	0.24303E 00
0.11906E 05	0.11871E 05	0.29647E 00
0.12249E 05	0.12170E 05	0.64721E 00
0.12712E 05	0.12781E 05	0.54066E 00

TABLE D - 12

PROCESSED DATA AND QUASILINEARIZATION RESULTS SAMPLE 112 RUN 3

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/DAY)	TEMP (DEG F)	VISCOSITY (CP)	Z FACTOR
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0.9355E 02	0.8895E 02	0.5462E-01	0.6759E 02	0.1762E-01	0.9985E 00
0.9355E 02	0.7205E 02	0.2220E 00	0.6793E 02	0.1762E-01	0.9987E 00
0.9355E 02	0.5955E 02	0.3169E 00	0.6831E 02	0.1762E-01	0.9988E 00
0.9355E 02	0.4805E 02	0.3822E 00	0.6859E 02	0.1762E-01	0.9988E 00
0.9354E 02	0.3654E 02	0.4383E 00	0.6877E 02	0.1762E-01	0.9989E 00
0.9354E 02	0.2654E 02	0.4721E 00	0.6879E 02	0.1762E-01	0.9990E 00
0.9354E 02	0.1364E 02	0.4999E 00	0.6900E 02	0.1762E-01	0.9991E 00

	K (MD)	B (PSIA)	FB (1/FT)
INITIAL GUESS	0.1661E 02	0.3815E 01	0.9235E 09
ITERATION NO. 1	0.1076E 02	0.2097E 01	0.4344E 10
ITERATION NO. 2	0.1210E 02	0.2096E 01	0.4344E 10
ITERATION NO. 3	0.1228E 02	0.2097E 01	0.4344E 10
ITERATION NO. 4	0.1229E 02	0.2097E 01	0.4343E 10
ITERATION NO. 5	0.1229E 02	0.2098E 01	0.4344E 10
ITERATION NO. 6	0.1229E 02	0.2098E 01	0.4344E 10

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TABLE D - 12

OBS.FUNCTION	CALC.FUNCTION	REL.ERR.(P.CENT)
0.83955E 03	0.83805E 03	0.17824E 00
0.35606E 04	0.35680E 04	0.20658E 00
0.52058E 04	0.52223E 04	0.31816E 00
0.64433E 04	0.63986E 04	0.69325E 00
0.74146E 04	0.74329E 04	0.24607E 00
0.80455E 04	0.80550E 04	0.11816E 00
0.85638E 04	0.85589E 04	0.57853E-01

TABLE D - 13

PROCESSED DATA AND QUASILINEARIZATION RESULTS SAMPLE 2 RUN 1

P(EXT) (PSIA)		P(WELL) (PSIA)		FLOW RATE (MSCF/DAY)	TEMP (DEG F)		VISCOSITY (CP)	Z FACTOR	
0.2135E	03	0.1359E	02	0.6817E-01	0.6850E	02	0.1767E-01	0.9982E	00
0.1935E	03	0.1359E	02	0.5535E-01	0.6865E	02	0.1766E-01	0.9984E	00
0.1735E	03	0.1359E	02	0.4392E-01	0.6875E	02	0.1765E-01	0.9985E	00
0.1535E	03	0.1359E	02	0.3381E-01	0.6884E	02	0.1764E-01	0.9987E	00
0.1335E	03	0.1359E	02	0.2520E-01	0.6881E	02	0.1763E-01	0.9988E	00
0.1135E	03	0.1358E	02	0.1780E-01	0.6881E	02	0.1762E-01	0.9990E	00
0.9358E	02	0.1358E	02	0.1192E-01	0.6836E	02	0.1760E-01	0.9991E	00
0.7358E	02	0.1358E	02	0.7380E-02	0.6868E	02	0.1760E-01	0.9993E	00
0.6158E	02	0.1358E	02	0.5250E-02	0.6854E	02	0.1759E-01	0.9993E	00
0.4958E	02	0.1358E	02	0.3420E-02	0.6838E	02	0.1758E-01	0.9994E	00
0.4158E	02	0.1358E	02	0.2370E-02	0.6822E	02	0.1757E-01	0.9995E	00
0.3558E	02	0.1358E	02	0.1680E-02	0.6834E	02	0.1757E-01	0.9995E	00
0.2957E	02	0.1357E	02	0.1120E-02	0.6813E	02	0.1756E-01	0.9996E	00
0.2158E	02	0.1358E	02	0.4800E-03	0.6813E	02	0.1755E-01	0.9997E	00

		K (MD)	B (PSIA)	FB (1/FT)
INITIAL GUESS		0.8976E 00	0.1043E 01	0.3429E 11
ITERATION NO.	1	0.6746E 00	0.9196E 00	-0.7662E 11
ITERATION NO.	2	0.7162E 00	0.9196E 00	-0.7661E 11
ITERATION NO.	3	0.7189E 00	0.9196E 00	-0.7662E 11
ITERATION NO.	4	0.7189E 00	0.9196E 00	-0.7661E 11

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TABLE D - 13

OBS.FUNCTION	CALC.FUNCTION	REL.ERR.(P.CENT)
0.45435E 05	0.45329E 05	0.23400E 00
0.37292E 05	0.37400E 05	0.28967E 00
0.29948E 05	0.30083E 05	0.44825E 00
0.23405E 05	0.23424E 05	0.83937E-01
0.17661E 05	0.17612E 05	0.27861E 00
0.12716E 05	0.12522E 05	0.15197E 01
0.85728E 04	0.84069E 04	0.19357E 01
0.52296E 04	0.52184E 04	0.21338E 00
0.36081E 04	0.37096E 04	0.28116E 01
0.22741E 04	0.24113E 04	0.60350E 01
0.15447E 04	0.16664E 04	0.78732E 01
0.10817E 04	0.11786E 04	0.89536E 01
0.69041E 03	0.78317E 03	0.13434E 02
0.28136E 03	0.33375E 03	0.18618E 02

TABLE D - 14

PROCESSED DATA AND QUASILINEARIZATION RESULTS SAMPLE 2 RUN 2

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/DAY)	TEMP (DEG F)	VISCOSITY (CP)	Z FACTOR
0.2136E 03	0.1966E 03	0.1099E-01	0.6987E 02	0.1780E-01	0.9971E 00
0.2136E 03	0.1736E 03	0.2291E-01	0.7013E 02	0.1780E-01	0.9973E 00
0.2136E 03	0.1556E 03	0.3085E-01	0.7002E 02	0.1779E-01	0.9974E 00
0.2136E 03	0.1216E 03	0.4330E-01	0.6863E 02	0.1773E-01	0.9975E 00
0.2136E 03	0.9420E 02	0.5136E-01	0.6847E 02	0.1771E-01	0.9977E 00
0.2136E 03	0.6960E 02	0.5680E-01	0.6836E 02	0.1770E-01	0.9978E 00
0.2136E 03	0.4360E 02	0.6103E-01	0.6825E 02	0.1768E-01	0.9980E 00
0.2136E 03	0.2760E 02	0.6329E-01	0.6825E 02	0.1767E-01	0.9981E 00
0.2136E 03	0.1360E 02	0.6466E-01	0.6822E 02	0.1766E-01	0.9982E 00

K (MD)	B (PSIA)	FB (1/FT)
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INITIAL GUESS		0.8976E 00	0.1043E 01	0.3429E 11
ITERATION NO.	1	0.6857E 00	0.3595E 02	0.2717E 12
ITERATION NO.	2	0.7239E 00	0.3595E 02	0.2717E 12
ITERATION NO.	3	0.7261E 00	0.3595E 02	0.2717E 12
ITERATION NO.	4	0.7261E 00	0.3595E 02	0.2717E 12
ITERATION NO.	5	0.7261E 00	0.3595E 02	0.2717E 12

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TABLE D - 14

OBS.FUNCTION	CALC.FUNCTION	REL.ERR.(P.CENT)
0.69734E 04	0.71793E 04	0.29535E 01
0.15488E 05	0.15501E 05	0.86504E-01
0.21413E 05	0.21326E 05	0.40828E 00
0.30838E 05	0.30674E 05	0.53148E 00
0.36751E 05	0.36895E 05	0.39244E 00
0.40780E 05	0.40870E 05	0.21966E 00
0.43724E 05	0.43578E 05	0.33288E 00
0.44863E 05	0.44925E 05	0.13964E 00
0.45439E 05	0.45440E 05	0.82634E-03

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PROCESSED DATA AND QUASILINEARIZATION RESULTS SAMPLE 2 RUN 3

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/DAY)	TEMP (DEG F)	VISCOSITY (CP)	Z FACTOR
0.2360E 02	0.1360E 02	0.6700E-03	0.7063E 02	0.1762E-01	0.9997E 00
0.2961E 02	0.1361E 02	0.1200E-02	0.7095E 02	0.1763E-01	0.9996E 00
0.3362E 02	0.1362E 02	0.1630E-02	0.7002E 02	0.1761E-01	0.9996E 00
0.3560E 02	0.1360E 02	0.1840E-02	0.7038E 02	0.1762E-01	0.9996E 00
0.4162E 02	0.1362E 02	0.2530E-02	0.7004E 02	0.1761E-01	0.9995E 00
0.5360E 02	0.1360E 02	0.4260E-02	0.7027E 02	0.1763E-01	0.9994E 00
0.7360E 02	0.1360E 02	0.8090E-02	0.6993E 02	0.1763E-01	0.9993E 00
0.9365E 02	0.1365E 02	0.1284E-01	0.6913E 02	0.1762E-01	0.9991E 00
0.1136E 03	0.1365E 02	0.1863E-01	0.6921E 02	0.1763E-01	0.9990E 00
0.1336E 03	0.1365E 02	0.2538E-01	0.6934E 02	0.1765E-01	0.9988E 00
0.1536E 03	0.1365E 02	0.3293E-01	0.6939E 02	0.1766E-01	0.9987E 00
0.1736E 03	0.1365E 02	0.4165E-01	0.6956E 02	0.1767E-01	0.9985E 00
0.1936E 03	0.1365E 02	0.5112E-01	0.6960E 02	0.1769E-01	0.9984E 00
0.2136E 03	0.1365E 02	0.6146E-01	0.6967E 02	0.1770E-01	0.9983E 00

K
(MD)B
(PSIA)FB
(1/FT)

INITIAL GUESS		0.8976E 00	0.1043E 01	0.3429E 11
ITERATION NO.	1	0.6470E 00	0.6884E 01	0.4652E 11
ITERATION NO.	2	0.6974E 00	0.6884E 01	0.4653E 11
ITERATION NO.	3	0.7016E 00	0.6884E 01	0.4653E 11
ITERATION NO.	4	0.7017E 00	0.6884E 01	0.4652E 11

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TABLE D - 15

OBS.FUNCTION	CALC.FUNCTION	REL.ERR.(P.CENT)
0.37219E 03	0.36543E 03	0.18172E 01
0.69183E 03	0.68223E 03	0.13863E 01
0.94498E 03	0.94727E 03	0.24184E 00
0.10828E 04	0.10791E 04	0.33657E 00
0.15469E 04	0.15139E 04	0.21370E 01
0.26887E 04	0.26548E 04	0.12637E 01
0.52331E 04	0.52741E 04	0.78189E 00
0.85846E 04	0.85918E 04	0.83765E-01
0.12730E 05	0.12756E 05	0.19890E 00
0.17676E 05	0.17710E 05	0.19184E 00
0.23423E 05	0.23344E 05	0.33556E 00
0.29969E 05	0.29989E 05	0.68909E-01
0.37315E 05	0.37318E 05	0.87655E-02
0.45461E 05	0.45467E 05	0.12225E-01

TABLE D - 16

PROCESSED DATA AND QUASILINEARIZATION RESULTS SAMPLE 2 RUN 4

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/DAY)	TEMP (DEG F)	VISCOSITY (CP)	Z FACTOR
0.2134E 03	0.1349E 02	0.6206E-01	0.6795E 02	0.1765E-01	0.9982E 00
0.2134E 03	0.2787E 02	0.6079E-01	0.6829E 02	0.1767E-01	0.9981E 00
0.2134E 03	0.4212E 02	0.5945E-01	0.6836E 02	0.1768E-01	0.9980E 00
0.2135E 03	0.6645E 02	0.5592E-01	0.6973E 02	0.1773E-01	0.9979E 00
0.2135E 03	0.1013E 03	0.4827E-01	0.6995E 02	0.1775E-01	0.9977E 00
0.2135E 03	0.1335E 03	0.3829E-01	0.7018E 02	0.1778E-01	0.9975E 00
0.2135E 03	0.1470E 03	0.3298E-01	0.7189E 02	0.1783E-01	0.9975E 00
0.2135E 03	0.1795E 03	0.1858E-01	0.7218E 02	0.1785E-01	0.9974E 00

K (MD)	B (PSIA)	FB (1/FT)
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INITIAL GUESS		0.8976E 00	0.1043E 01	0.3429E 11
ITERATION NO.	1	0.7231E 00	0.3061E 01	0.7835E 11
ITERATION NO.	2	0.7504E 00	0.3060E 01	0.7835E 11
ITERATION NO.	3	0.7514E 00	0.3065E 01	0.7835E 11
ITERATION NO.	4	0.7514E 00	0.3062E 01	0.7836E 11
ITERATION NO.	5	0.7515E 00	0.3059E 01	0.7836E 11
ITERATION NO.	6	0.7515E 00	0.3063E 01	0.7835E 11
ITERATION NO.	7	0.7515E 00	0.3062E 01	0.7836E 11
ITERATION NO.	8	0.7514E 00	0.3062E 01	0.7836E 11

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TABLE D - 16

OBS.FUNCTION	CALC.FUNCTION	REL.ERR.(P.CENT)
0.45398E 05	0.45548E 05	0.32923E 00
0.44804E 05	0.44674E 05	0.28906E 00
0.43806E 05	0.43704E 05	0.23387E 00
0.41198E 05	0.41228E 05	0.72951E-01
0.35347E 05	0.35399E 05	0.14636E 00
0.27772E 05	0.27832E 05	0.21701E 00
0.23985E 05	0.23973E 05	0.49930E-01
0.13385E 05	0.13308E 05	0.57810E 00

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